

Annual Report 2010

Measurement and Information in Optics

MSM 6198959213

and

Center of Modern Optics

LC06007

The Annual Report 2009 covers all the scientific activities of the long-term projects supported by the Czech Ministry of Education MSM 6198959213 **Measurement and Information in Optics MIO 2005 - 2011** and LC06007 **Center of Modern Optics CMO 2006 - 2010**. Results of individual small teams described comprehensively on a single page are documenting the progress of our research in the fields of modern optics and quantum information processing. The project CMO has been extended till the end 2011. The team members dr. Petr Marek has been awarded by the V. Votruba Prize and M. Odehnal Prize for theoretical physics and doc. J. Fiurášek has been awarded by the Prize of Ministry of Education for Science.

Olomouc, January 2011

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Quantum information experiments based on fiber optics

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In this subproject we are focused on optical-fiber implementations of various procedures from the field of quantum information processing. It means that experimental setups are mainly built using optical fibers and fiber components.

This year we have finished an experiment started some time ago. We investigated how distinguishability of a “noise” particle degrades interference of the “signal” particle. The signal, represented by an equatorial state of a photonic qubit, is mixed with noise, represented by another photonic qubit, via linear coupling on the beam splitter. We have found the degradation of the “signal” photon interference depending on the degree of indistinguishability between “signal” and “noise” photon. When the photons are principally completely distinguishable but technically indistinguishable the visibility drops to the value $1/\sqrt{2}$. As the photons become more indistinguishable the maximal visibility increases and reaches the unit value for completely indistinguishable photons. We have examined this effect experimentally using setup with fiber optics two-photon Mach-Zehnder interferometer.

Besides, we have started a new experiment exploiting an electronic feed-forward to increase the success probability of programmable quantum phase gate. By means of an electronic feed-forward we can control the quantum state of a data qubit according to the result of measurement on an ancillary qubit. The ability to modify the output of the quantum gate correspondingly to the result of a previous measurement allows a substantial increase in the probability of success of the gate. Besides, such an

electronic feed-forward can be used in other quantum op-

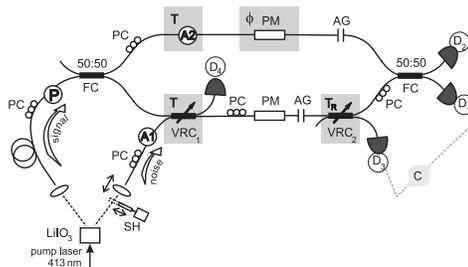


FIG. 1: Scheme of the experimental setup for post-selection reduction of noise.

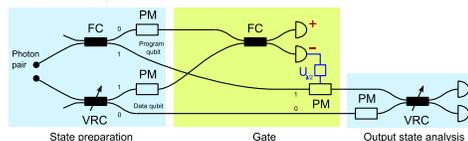


FIG. 2: Scheme of the programmable phase gate with an electronic feed-forward.

tics tasks. We have built a programmable quantum phase gate using fiber-optics components. It is based on two-photon interference in two interconnected Mach-Zehnder interferometers and we have incorporated our electronic feed-forward into this gate.

[1] M. Gavenda, L. Čelechovská, J. Soubusta, M. Dušek, R. Filip, *How much does a distinguishable particle destroy co-*

herence of a quantum bit?, submitted to Physical Review A.

Distillation and purification of continuous-variable quantum states of light and atoms

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Many different quantum-information communication protocols such as teleportation, dense coding, and entanglement-based quantum key distribution are based on the faithful transmission of entanglement between distant locations in an optical network. The distribution of entanglement in such a network is, however, hampered by loss and noise that is inherent in all practical quantum channels. Thus, to enable faithful transmission one must resort to the protocol of entanglement distillation. Motivated by this we have carried out a detailed theoretical analysis and an experimental realization of entanglement distillation for continuous-variable quantum states.

We have proposed an entanglement distillation and purification scheme for symmetric two-mode entangled Gaussian states that allows to asymptotically extract a pure entangled Gaussian state from any input entangled symmetric Gaussian state [1]. A key feature of this protocol is that it utilizes a two-copy deGaussification procedure that involves a Mach-Zehnder interferometer with single-mode non-Gaussian filters inserted in its two arms, see Fig. 1. In collaboration with the group of R. Schnabel in Hannover we have experimentally demonstrated the first iterative two-step distillation of entanglement. The output of a first distillation stage underwent a second distillation step and was made available for subsequent steps [2].

In collaboration with group of G. Leuchs in Erlangen we have performed detailed theoretical analysis and an experimental realization of continuous variable entanglement distillation in a channel that is inflicted by different kinds of non-Gaussian noise [3]. The continuous variable entangled states are sent through a free-space laboratory channel in which the losses are altered to simulate a free-space atmospheric channel with varying losses. We use linear optical components, homodyne measurements, and classical communication to distill the entanglement, and we find that by using this method the entanglement

can be probabilistically increased for some specific non-

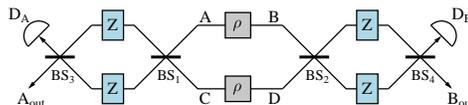


FIG. 1: Two-copy de-Gaussification scheme.

Gaussian noise channels.

We have also proposed a protocol for Gaussification of quantum states of traveling light beams in an atomic quantum memory that couples to light via quantum non-demolition (QND) interaction [4]. Importantly, the total required coupling strength scales only logarithmically with number of Gaussified light modes. The scheme can be used to prepare entangled states of two distant atomic ensembles and to purify and Gaussify noisy non-Gaussian entangled states of light while simultaneously storing the purified state in atomic memories.

Finally, we have proposed and experimentally demonstrated a universal quantum averaging process implementing the harmonic mean of quadrature variances [5]. The averaged variances are prepared probabilistically by means of linear optical interference and measurement-induced conditioning. We verify that the implemented harmonic mean yields a lower value than the corresponding value obtained for the standard arithmetic-mean strategy. The harmonic-mean protocol can be used to efficiently stabilize a set of squeezed-light sources with statistically fluctuating noise levels.

This work was supported by MSMT under the projects Measurement and Information in Optics (MSM 6198959213) and Center of Modern Optics (LC06007), by the Czech Science Foundation-GACR (202/07/J040) and by EU project COMPAS (212008).

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Advanced reconstruction schemes for analysis and extraction of information

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Angular performance measure for tighter uncertainty relations have been derived for conjugated angular momentum and angle variables. We propose an angular performance that allows for tighter uncertainty relations for angle and angular momentum. The differences with previous bounds can be significant for particular states and indeed may be amenable to experimental measurement with the present technology [1].

Non-negative Wigner functions for orbital angular momentum states has been also considered. The Wigner function of a pure continuous-variable quantum state is non-negative if and only if the state is Gaussian. Here we show that for the canonical pair angle and angular momentum, the only pure states with non-negative Wigner functions are the eigenstates of the angular momentum [2].

A comprehensive theory of the Weyl-Wigner formalism for the canonical pair angles-angular momentum has been presented, with special emphasis in the implications of rotational periodicity and angular-momentum discreteness [3]. The theory has been completed by experimental proposal concerning vortex tomography with nondiffracting beams, where information is encoded in a superposition of Bessel-like nondiffracting beams. The measurement of the angular probability distribution at different positions allows for the reconstruction of the Wigner function [4].

Operational definition of tomography based on fitting of data patterns has been proposed in Ref. [5] Each data pattern corresponds to the response of the measurement setup to a predefined reference state. The set of data patterns can be measured experimentally in the calibration stage preceding to the reconstruction. As the main advantage, the procedure is free of notorious problems with projections into non-normalizable quadrature eigen-

states, infinite dimensionality, ill-posed inversion, or imperfect detection.

The challenging scheme for quantum reconstruction of the mutual coherence function has been proposed in Ref. [6]. The current methods of light detection are sensitive both to robust features, such as intensity, or polarization, and to more subtle effects, such as correlations. Here we show how wave front detection, which allows for registering the direction of the incoming wave flux at a given position, can be used to reconstruct the mutual coherence function when combined with some techniques previously developed for quantum information processing.

This research was supported by the Research Project "Measurement and Information in Optics" MSM 6198959213.

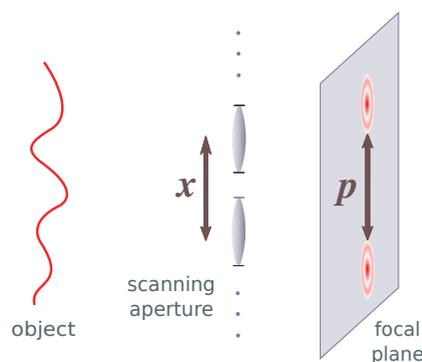


FIG. 1. Simplified scheme of a general scanning device. By moving the aperture, the field is sampled at different positions, whereas by moving the detector in the focal plane, the field is sampled at different angles [6].

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Non-classical states of light and quantum metrology

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Schrodinger cat state (SCS) which is of fundamental interest in quantum optics is given as a superposition of two or more classical coherent states which are indistinguishable and interfere with each other. For small amplitude of constituent coherent states the SCS can be prepared by subtracting a single photon from a squeezed vacuum state.

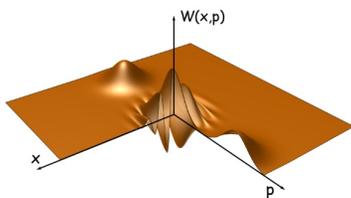


FIG. 1: Wigner function of the Schrodinger cat state shows oscillations and negative values revealing pure quantum interference. This highly non-classical state serves as an efficient resource for quantum information processing and quantum metrology.

The goal of the project is to prepare SCS with sufficiently high quality and demonstrate a quantum processor based on SCS. So far we have developed the source of pulsed squeezed light, SCS source [1] and conducted the first test of a quantum logic gate based on SCS.



FIG. 2: Photo of the experimental setup used for Schrodinger cat state generation and phase super-resolution.

Further, we have proposed and experimentally verified a novel phase super-resolution approach using classical resources with virtually no excess noise added. By probing a phase sample with coherent state with a mean number N of photons we are able to narrow the phase response function \sqrt{N} times below Rayleigh limit while retaining shot-noise limited sensitivity [2, 3].

The work is partially supported by the Research Project "Measurement and Information in Optics" MSM 6198959213 of the Czech Ministry of Education.

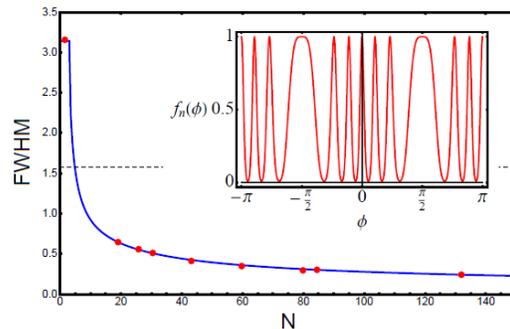


FIG. 3: Narrowing of the phase response (FWHM) and the typical response function over 2π (inset) which beats Rayleigh resolution limit \sqrt{N} times.

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Continuous-variable teleportation of a negative Wigner function

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Quantum teleportation [1] is a basic primitive for quantum communication [2, 3] and quantum computing [4]. We address the problem [5] of continuous-variable (unconditional and conditional) teleportation of a pure single-photon state and a mixed attenuated single-photon state generally in a nonunity gain regime. Our figure of merit is the maximum of negativity of the Wigner function in the origin which witnesses a highly non-classical feature of the teleported state. We find that negativity of the Wigner function of the single-photon state can be unconditionally teleported with arbitrarily weak squeezed state with variance V_{sq} used to create the entangled state shared in the teleportation (see Fig. 1).

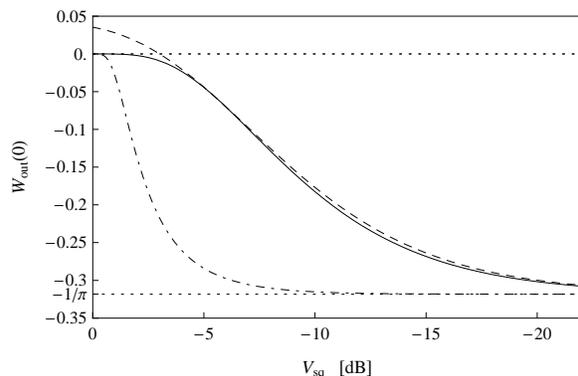


FIG. 1: Output Wigner function in the origin versus the squeezed variance V_{sq} for optimal nonunity gain teleportation (solid curve), unity gain teleportation (dashed curve) and optimal conditional teleportation where we accept only the outcomes of Bell measurement falling into the circle with radius $K = 0.3$ (dash-dotted curve) of a single-photon Fock state.

In contrast, Fig. 2 shows that for the attenuated single-photon state there is a strict threshold squeezing one has to surpass in order to successfully teleport negativity of its Wigner function. The conditional teleportation allows to approach perfect transmission of the single pho-

ton for an arbitrarily low squeezing at a cost of a success rate. On the other hand, for the attenuated single photon, it cannot overcome the squeezing threshold of unconditional teleportation and it can approach maximal negativity only if the squeezing increases simultaneously. However, once the threshold squeezing is surpassed the conditional teleportation pronouncedly outperforms the unconditional one. The main consequences for quantum communication and quantum computing with continuous variables are discussed.

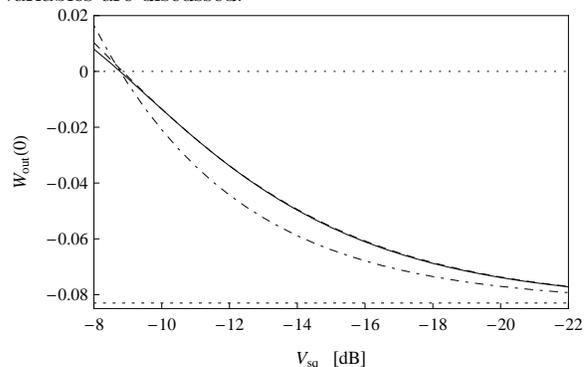


FIG. 2: Output Wigner function in the origin versus the squeezed variance V_{sq} for optimal nonunity gain teleportation (solid curve), unity gain teleportation (dashed curve) and optimal conditional teleportation with $K = 0.3$ (dash-dotted curve) for the input state $\eta|1\rangle\langle 1| + (1 - \eta)|0\rangle\langle 0|$ with $\eta = 0.6304$. Bottom dotted curve corresponds to the input Wigner function in the origin $W_{in}^{(\eta)}(0) = -0.083$.

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Noiseless amplification of coherent states of light

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Light is a natural carrier of information, no matter if we are talking about the strong signals, used in optical fibres worldwide, or about weak signals, employed by the recently put forth quantum communication protocols, such as quantum key distribution. One problem, quantum communication needs to deal with, is the problem of losses. In classical communication, losses can be compensated by amplifying the signal, an approach that does not work particularly well for quantum signals, because traditional means of amplification incur a prohibitive noise penalty. However, that does not mean it is wrong to amplify, just that another means of doing it need to be devised.

We have theoretically proposed two amplifiers for weak coherent states of light [1]. Both the amplifiers allow probabilistic amplification of a weak coherent state without introducing the hampering noise, which appears in deterministic amplifications. The amplifiers employ high order nonlinearities represented by single photon operations.

The first approach to amplification, which relies on controlled addition and subtraction of photons into the signal mode of light, see Fig. 1, allows for high fidelity amplification, as was confirmed by the experimental verification of the proposal.

The second proposal does not rely on the expensive single photon addition. Instead it utilizes rather counter-intuitive noise injection, when the signal is intentionally made more noisy, so it can be purified and simultaneously amplified by the subsequent photon subtraction, see Fig. 2. This interesting primitive was, with our participation, also verified experimentally [2].

In the upcoming year we plan to keep investigating

the amplifier, its properties and possible applications. Namely, we are going to focus at using the amplifier to implement a phase covariant cloner of coherent state, and to analyze its suitability for quantum cryptography and quantum metrology protocols.

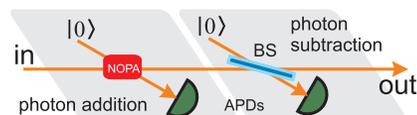


FIG. 1: Experimental setup of the high-fidelity amplifier for coherent states.



FIG. 2: Experimental setup of the proposed noise-powered amplifiers for coherent states.

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Digital 3D image reconstruction in FINCH

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Recently, Fresnel Incoherent Correlation Holography (FINCH) has been proposed as perspective method for 3D digital imaging of incoherently illuminated objects [1]. In FINCH, holograms of the incoherent object are created by means of wavefront splitting realized by the Spatial Light Modulator (SLM). Subsequently, three different CCD records of the object are processed and 3D image is digitally reconstructed applying convenient algorithms.

In the project, the paraxial computational model of FINCH was proposed and used for detailed analysis of 3D digital image. As the main result, dependence of the lateral and longitudinal image magnifications of the digital image on set-up parameters was exactly described and used for design of modified variants of FINCH experiment [2]. Predictions of the theoretical model were verified ex-

perimentally transverse and longitudinal scale of the 3D digital image and to achieve imaging with object-space and image-space telecentricity. To verify theoretical predictions, the FINCH imaging with a scanning point source was realized. As a point source, the single mode fiber with the holder mounted to the XYZ travel translation stages was used. The wavefront splitting was realized by the reflective SLM Hamamatsu X10468 (16x12 mm², 792x600 px) operating in a pure phase mode and intensity records of the object were performed with CCD camera Olympus F-View II (9.9x6.7 mm², 1376x1032 px). In the experiment, the lateral and longitudinal magnifications of the digital image were determined from experimental data for appropriately chosen parameters of FINCH set-up and compared with results of the theoretical model. The image point used

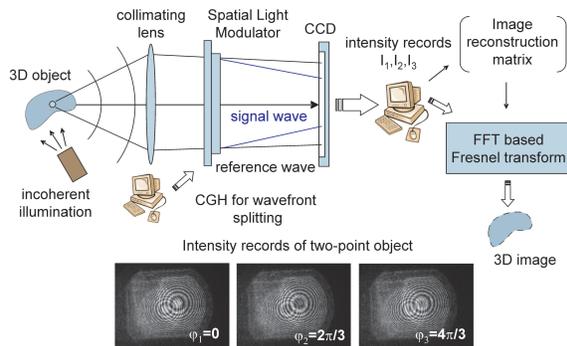


FIG. 1: Illustration of 3D digital imaging in FINCH experiment.

perimentally. Principle of FINCH digital imaging is illustrated in Fig.1. The 3D object is placed near the focal plane of the collimating lens and illuminated by incoherent source. The light scattered by the object is collimated and directed towards the SLM. At the SLM addressed by the Computer Generated Hologram (CGH), the light wave originating from a single point of the spatially incoherent object is split into signal and reference waves. If the optical path difference of the waves is less than the coherence length of the light source, the waves interfere and the created interference pattern is captured by CCD. In FINCH, three records of the object with different phase shifts of the reference wave are performed to eliminate holographic twin image. Subsequently, the image reconstruction matrix is created and the object is digitally reconstructed from CCD records applying FFT based Fresnel transform (Fig.2). The theoretical model of FINCH proposed in [2] demonstrates a possibility to control indepen-

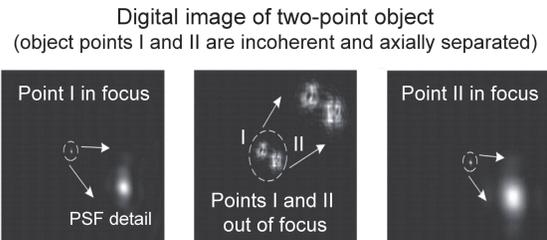


FIG. 2: Digital image of two incoherent object points.

in the calculation of the lateral magnification was defined as the center of gravity of the intensity spot created in digital reconstruction. The longitudinal coordinate of the digital image was defined as the position where the reconstructed image has an optimal transverse intensity profile. The longitudinal localization of the image was ensured by the numerical procedure enabling calculation of the first- and second-order moments and determination of the standard deviations of the image intensity distribution. From the optimal image positions obtained for the separate on-axis positions of the object point, the longitudinal magnification was determined. The experimental results validated theoretical parameters of FINCH imaging and feasibility of variants of experimental set-up adapted to requirements of special applications.

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Qubit-induced phonon blockade as a signature of quantum behavior in nanomechanical resonators

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The observation of quantized nanomechanical oscillations by detecting femtometer-scale displacements is a significant challenge for experimentalists. We propose [1] that phonon blockade can serve as a signature of quantum behavior in nanomechanical resonators. The main idea for phonon blockade is that the second phonon cannot be excited when there is one phonon in the nonlinear oscillator. To realize phonon blockade, a superconducting quantum two-level system is coupled to the nanomechanical resonator and is used to induce the phonon self-interaction. We show how the oscillation of the resonator can occur in the quantum regime and demonstrate how the phonon blockade can be observed with currently accessible experimental parameters.

The coupling between a nanomechanical resonator and superconducting charge qubit shown in Fig. 1 can be described by the effective Hamiltonian [1]

$$H_{\text{eff}} = \hbar\bar{\omega}a^\dagger a + \hbar\kappa a^\dagger a(a^\dagger a - 1) + \hbar\epsilon(a^\dagger e^{-i\omega_2 t} + a e^{i\omega_2 t}) \quad (1)$$

with a renormalized frequency $\bar{\omega} = \omega + \kappa - g^2/\Delta$.

In the case the *phonon self-interaction* strength κ is much larger than the phonon decay rate γ the oscillation of the NAMR is in the quantum regime, the phonon transmission *can be blocked* in analogy to the single-photon blockade in a cavity. It is easy to see that the two states $|0\rangle$ and $|1\rangle$ with zero eigenvalues are degenerate in the term $\kappa a^\dagger a(a^\dagger a - 1)$ of Eq. (1). This degeneracy plays a crucial role in the phonon blockade. Indeed, if we assume that the interaction strength ϵ is much smaller than the nonlinearity constant κ (i.e., $\epsilon \ll \kappa$), then the phonon eigenstates of the Hamiltonian (1) can become a superposition of only two states, $|0\rangle$ and $|1\rangle$, in the lowest-order approximation of the expansion in the strength ϵ .

As shown in Fig. 1, the induced electromotive force V between two ends of the NAMR is

$$V = BL \frac{p}{m} = iBL \sqrt{\frac{\hbar\omega}{2m}} (a^\dagger - a) \quad (2)$$

which can be experimentally measured. Here, p is the momentum for the center of the NAMR mass. We have analyzed numerically the power spectrum

$$S_V(\omega') = \int_{-\infty}^{\infty} \langle V(0)V(\tau) \rangle e^{-i\omega'\tau} d\tau \quad (3)$$

defined by the Fourier transform of the induced electromotive-force two-time correlation function. This power spectrum can be measured effectively.

Assuming perfect phonon blockade, i.e., truncation to an exact qubit state, one can analyze the whole evolution

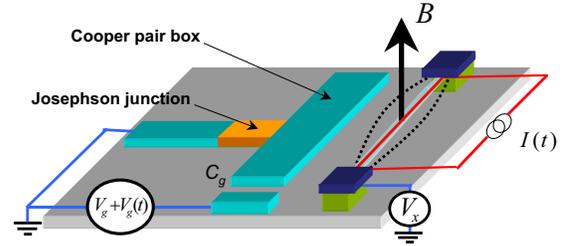


FIG. 1: Schematic diagram for the coupling between a nanomechanical resonator (light blue bar on the right side) and superconducting charge qubit (left side). Two black dashed curves on the right show that the resonator is oscillating. The static magnetic field B (presented by the black upward-pointing arrow) and the alternating current $I(t)$ (shown by the red loop on the right) are used for the motion detection of the NAMR.

of our system confined in two-dimensional Hilbert space and find that the corresponding power spectrum should have at most three peaks at frequencies $\omega' \approx 0, \pm 2\epsilon$. It is seen that these frequencies are independent of κ . A peak at $\omega'_0 = 0$ does not appear for real ϵ , which is the case analyzed in the paper. In contrast, four new peaks $\omega' \approx \pm(2\kappa \pm \epsilon)$ appear in the spectrum in the case of not perfect phonon blockade. This can be obtained numerically and understood by analyzing a Hilbert space of dimension $d > 2$.

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[1] Yu-xi Liu, A. Miranowicz, Y. B. Gao, J. Bajer, C. P. Sun, and F. Nori, Phys. Rev. A **82** (2010) 032101