Control of decoherence

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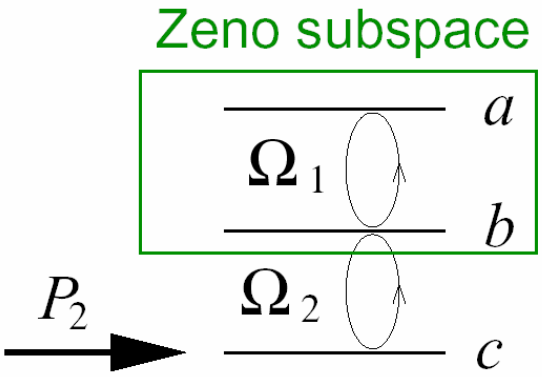
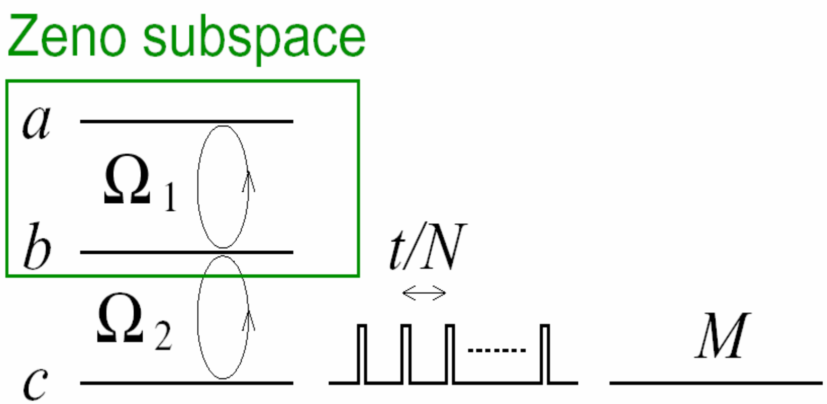
Olomouc,
20 October 2005



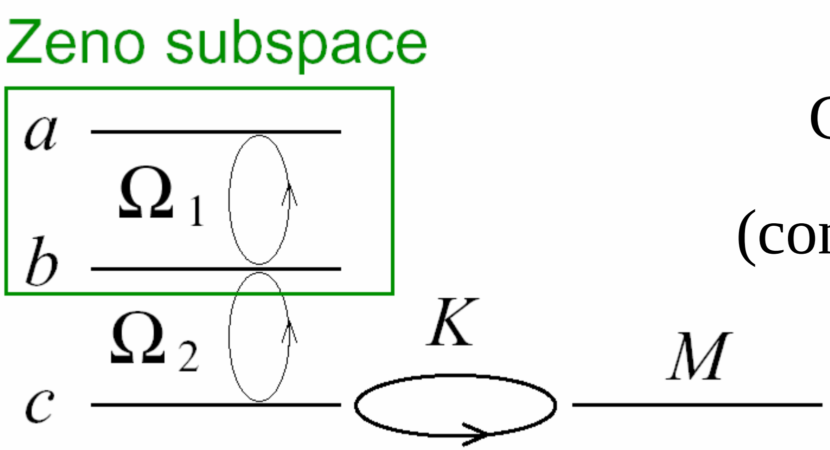
Measurements

(projections)



Unitary kicks

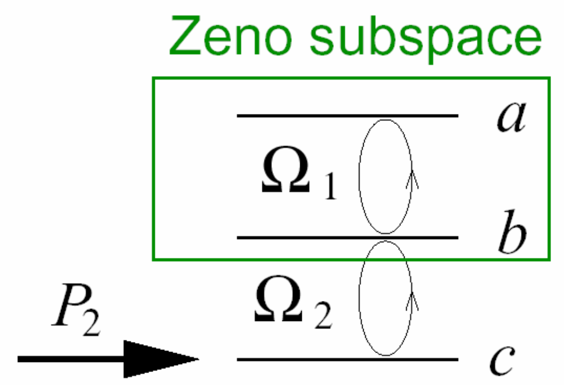
(“bang bang”)



Continuous coupling

(continuous measurement)

Projections



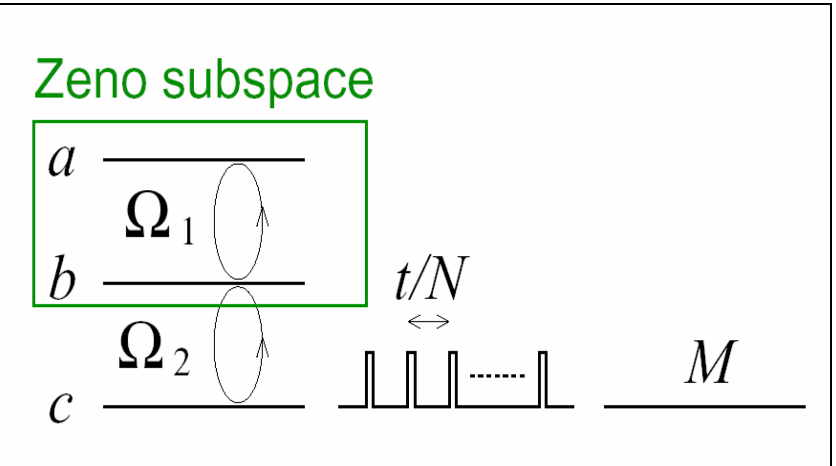
$$P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H = \begin{pmatrix} 0 & \Omega_1 & 0 \\ \Omega_1 & 0 & \Omega_2 \\ 0 & \Omega_2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & \Omega_1 & 0 \\ \Omega_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = H_Z$$

$\Uparrow$

Zeno limit $(N \rightarrow \infty)$

Kicks



$$U_{\text{kick}} = \begin{pmatrix} \exp(-i\lambda_1) & 0 & 0 & 0 \\ 0 & \exp(-i\lambda_1) & 0 & 0 \\ 0 & 0 & \cos\lambda_2 & -i\sin\lambda_2 \\ 0 & 0 & -i\sin\lambda_2 & \cos\lambda_2 \end{pmatrix}$$

$$H = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & \Omega_2 & 0 \\ 0 & \Omega_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = H_Z$$

$\Uparrow$

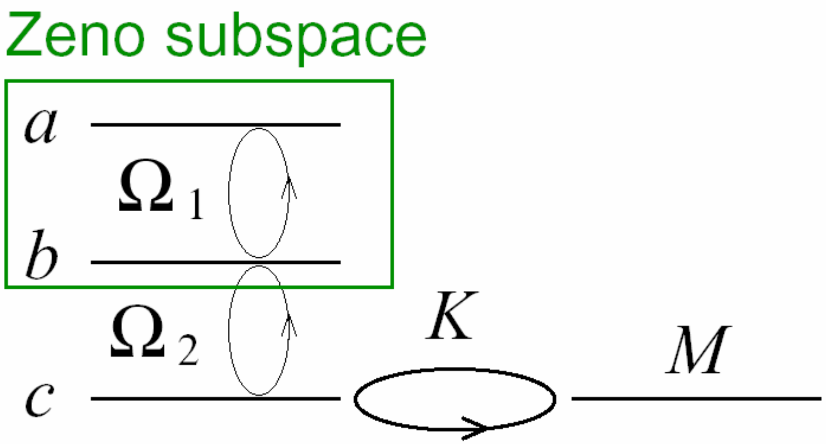
Zeno limit $(N \to \infty)$

Itano, Heinzen, Bollinger, Wineland (1990)

Continuous coupling

Simonius 1978

Harris & Stodolsky 1982



$$K H_c = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K \\ 0 & 0 & K & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & \Omega_2 & 0 \\ 0 & \Omega_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = H_Z$$

$\Uparrow$

Zeno limit

$$(K \rightarrow \infty)$$

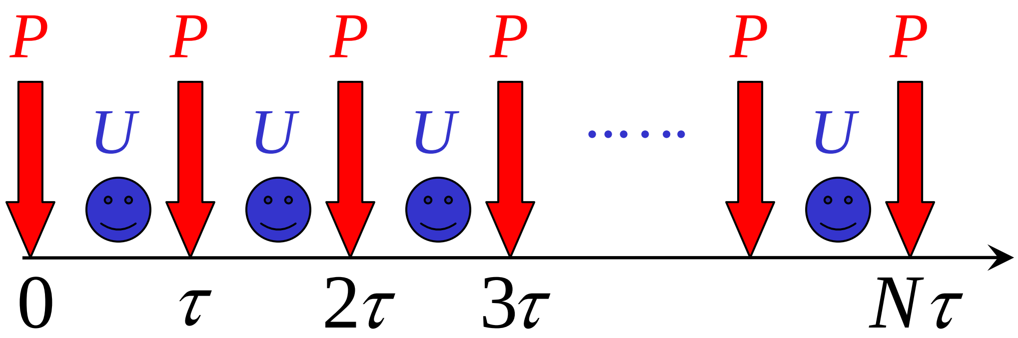
Superselection rule

In the *same sense* as Wick, Wightman and Wigner

(Sudarshan's comment)

Quantum Zeno effect

Prepare Q in the initial state ρ_0 at time 0 and perform a series of P-observations at times $\tau_j = jt/N = j\tau, \quad (j = 1, \ldots, N)$.



$$\rho^{(N)}(t) = V_N(t)\rho_0 V_N(t)^\dagger, \qquad V_N(t) = [PU(t/N)P]^N$$

The probability to find the system in $\mathcal{H}_P$ reads

$$p^{(N)}(t) = \mathrm{Tr}[V_N(t)\rho_0 V_N(t)^\dagger] \xrightarrow{N\to\infty} \mathrm{Tr}[P\rho_0] = 1$$

Quantum Zeno effect: repeated observation in succession inhibit transitions outside $\mathcal{H}_P$

Misra and Sudarshan 1977

Nonselective measurements

Partition $\mathcal{H} = \bigoplus_n \mathcal{H}_{P_n}, \qquad \sum_n P_n = 1$

Free evolution $\hat{U}_t \rho = U(t)\rho U(t)^\dagger, \quad U(t) = \exp(-iHt)$

Nonselective measurement

Schwinger, Proc. Nat. Acad. Sc. 45, 1552 (1959)

$$\hat{P}\rho = \sum_n P_n \rho P_n$$

Zeno evolution

$$\hat{V}_Z(t) = \lim_{N\to\infty} \hat{V}^{(N)}(t) = \lim_{N\to\infty} \left(\hat{P}\hat{U}_{t/N}\right)^N$$

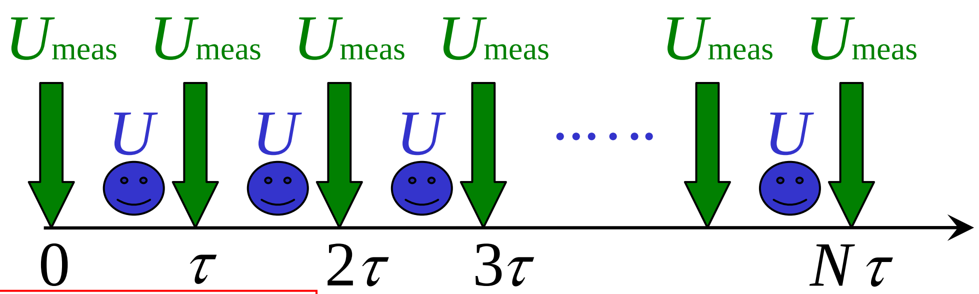
$$\hat{V}_Z(t)\rho = \sum_n V_n(t)\rho V_n(t)^\dagger, \quad V_n(t) = P_n \exp(-iP_n H P_n t)$$

Unitary kicks

Q undergoes N kicks U_{meas} (instantaneous unitary transformations) in a time interval t $\quad (P \rightarrow U_{\text{meas}})$

$$V_N(t) = \left[U_{\text{meas}} U\left(\frac{t}{N}\right) \right]^N,$$

$$U(t) = \exp(-iHt)$$



Limiting evolution $N \rightarrow \infty$

$$V_Z(t) = U_{\text{meas}}^N \exp(-iH_Z t)$$

$$H_Z = \sum_n P_n H P_n, \qquad U_{\text{meas}} = \sum_n e^{-i\lambda_n} P_n$$

$$V_Z(t) = \sum_n P_n \exp(-iN\lambda_n - iP_n H P_n t)$$

Identical to quantum maps in quantum chaos

Casati, Chirikov, Ford and Izrailev (1979)
Berry and Balazs (1979)

Dynamical decoupling
"Bang-bang" control

Viola, Knill, Lloyd, PRL (1999)
Byrd, Lidar, Quant. Inf. Proc. (2002)
Facchi, Tasaki, Pascazio, Nakazato, Lidar, PRA (2005)

Continuous coupling

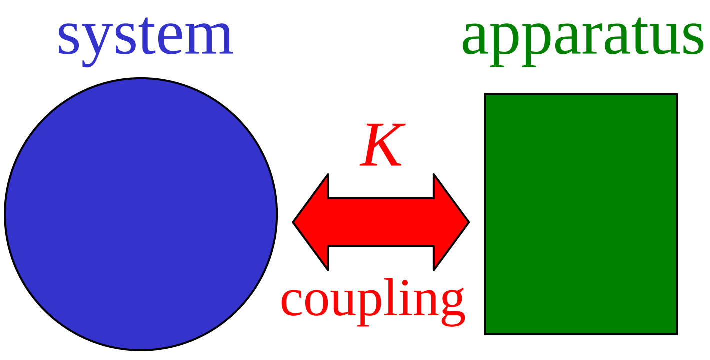
Observed system + apparatus

$$H_K = H + KH_{\text{meas}}, \qquad V_K = \exp(-iH_K t)$$

H system Hamiltonian (+ apparatus free Hamiltonian)

H_{meas} interaction between system and apparatus

K strength of the "measurement".



Strong measurement limit $K \to \infty$

$$V_Z(t) = \exp(-iKH_{\text{meas}}t)\exp(-iH_Z t)$$

$$H_Z = \sum_n P_n H P_n, \qquad H_{\text{meas}} = \sum_n \lambda_n P_n$$

$$V_Z(t) = \sum_n P_n \exp(-iK\lambda_n t - iP_n H P_n t)$$

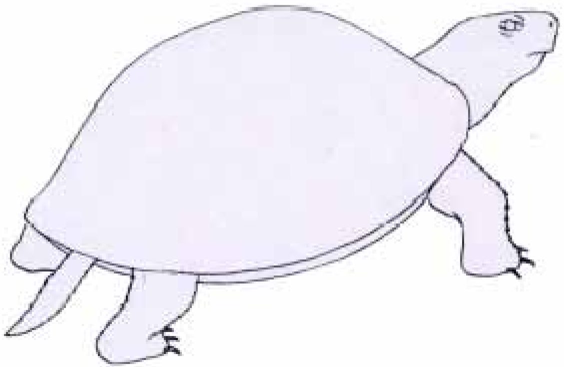
Facchi and Pascazio, Phys. Rev. Lett. 89, 080401(2002)

Comparison

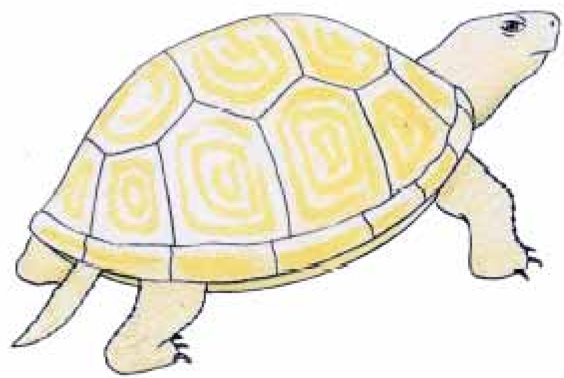
$$\hat{V}_Z(t)\rho = \sum_n V_n(t)\rho V_n(t)^\dagger, \quad V_n(t) = P_n \exp(-i P_n H P_n t)$$

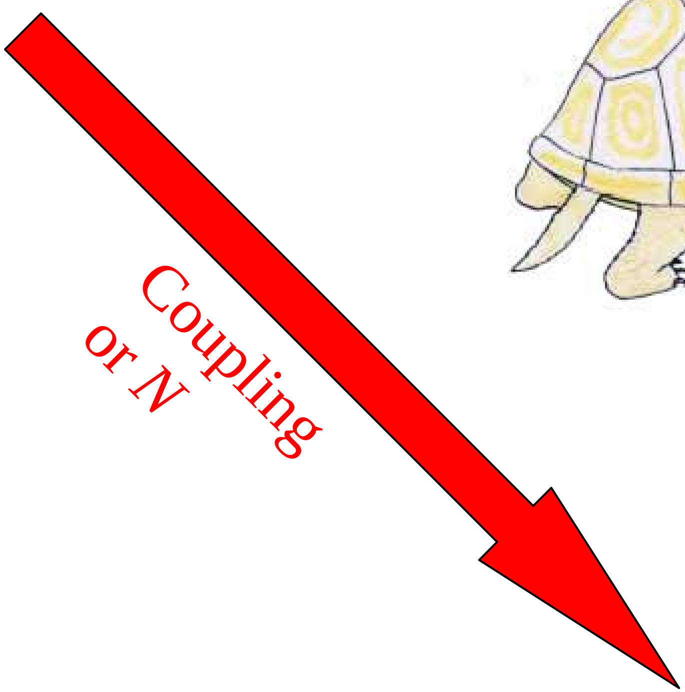
$$V_Z(t) = \sum_n P_n \exp(-iN\lambda_n - iP_n H P_n t)$$

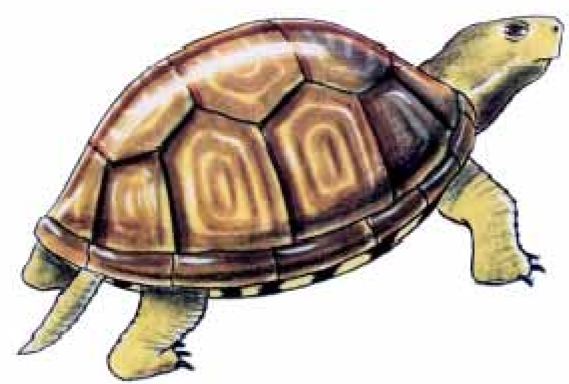
$$V_Z(t) = \sum_n P_n \exp(-iK\lambda_n t - iP_n H P_n t)$$



Dynamical superselection sectors







Objective: understand and suppress decoherence

Palma, Suominen and Ekert (1996)
Duan and Guo (1997)
Zanardi and Rasetti (1997)
Lidar, Chuang and Whaley (1998)
Viola, Knill and Lloyd (1999)
Vitali and Tombesi (1999, 2001)
Beige, Braun, Tregenna and Knight (2000)
…

Facchi, Lidar and Pascazio (2003)
Facchi, Tasaki, Pascazio, Nakazato, Tokuse, Lidar (2005)

Benenti, Casati, Montangero, Shepelyansky (2001)

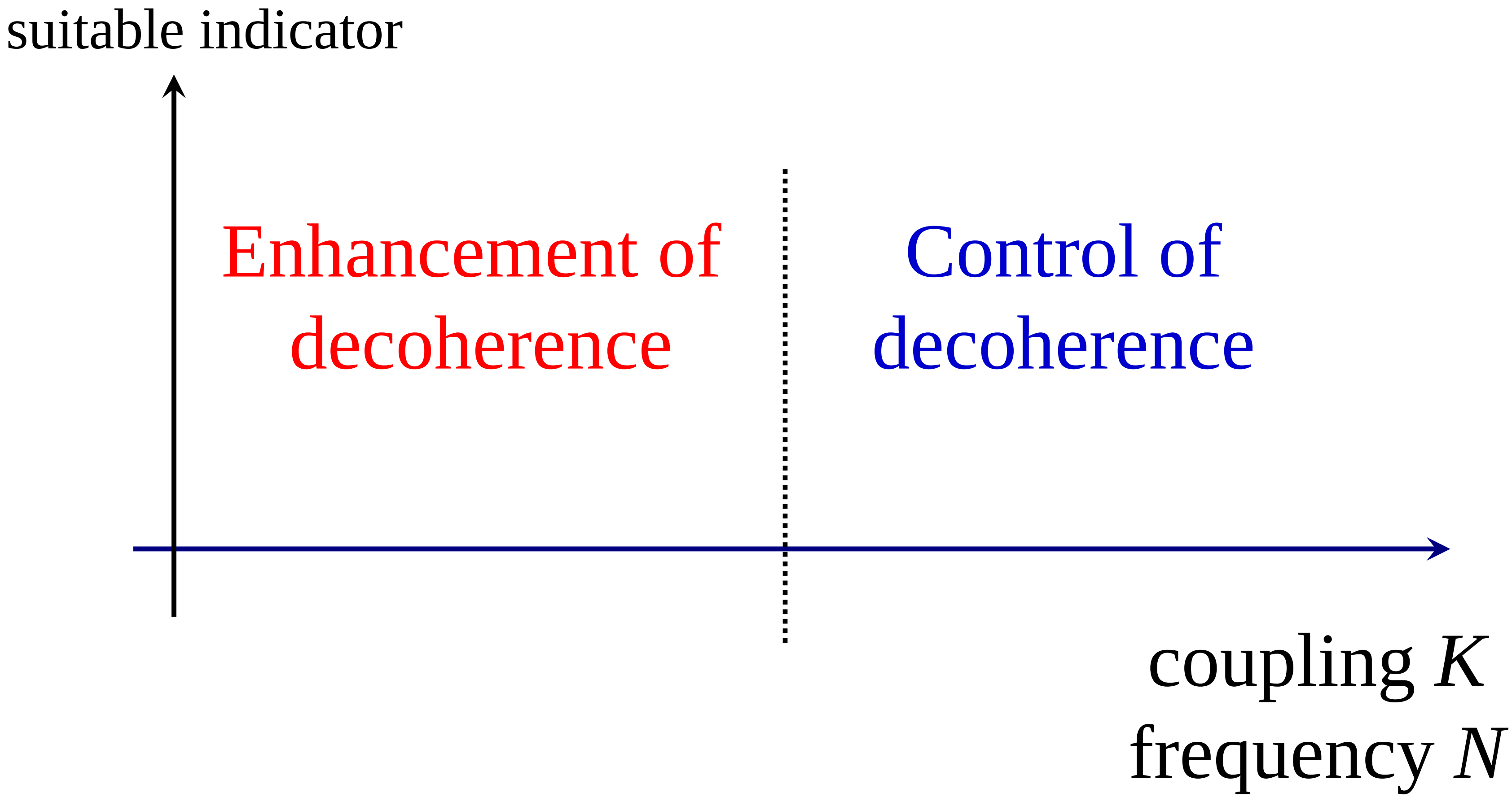
Vitali, Tombesi, Milburn (1997, 1998)
Fortunato, Raimond, Tombesi, Vitali (1999)
Kofman, Kurizki (2001)
Agarwal, Scully, Walther (2001)

Calarco, Datta, Fedichev, Pazy, Zoller,
"Spin-based all-optical quantum computation with quantum dots: understanding and suppressing decoherence" (2003)

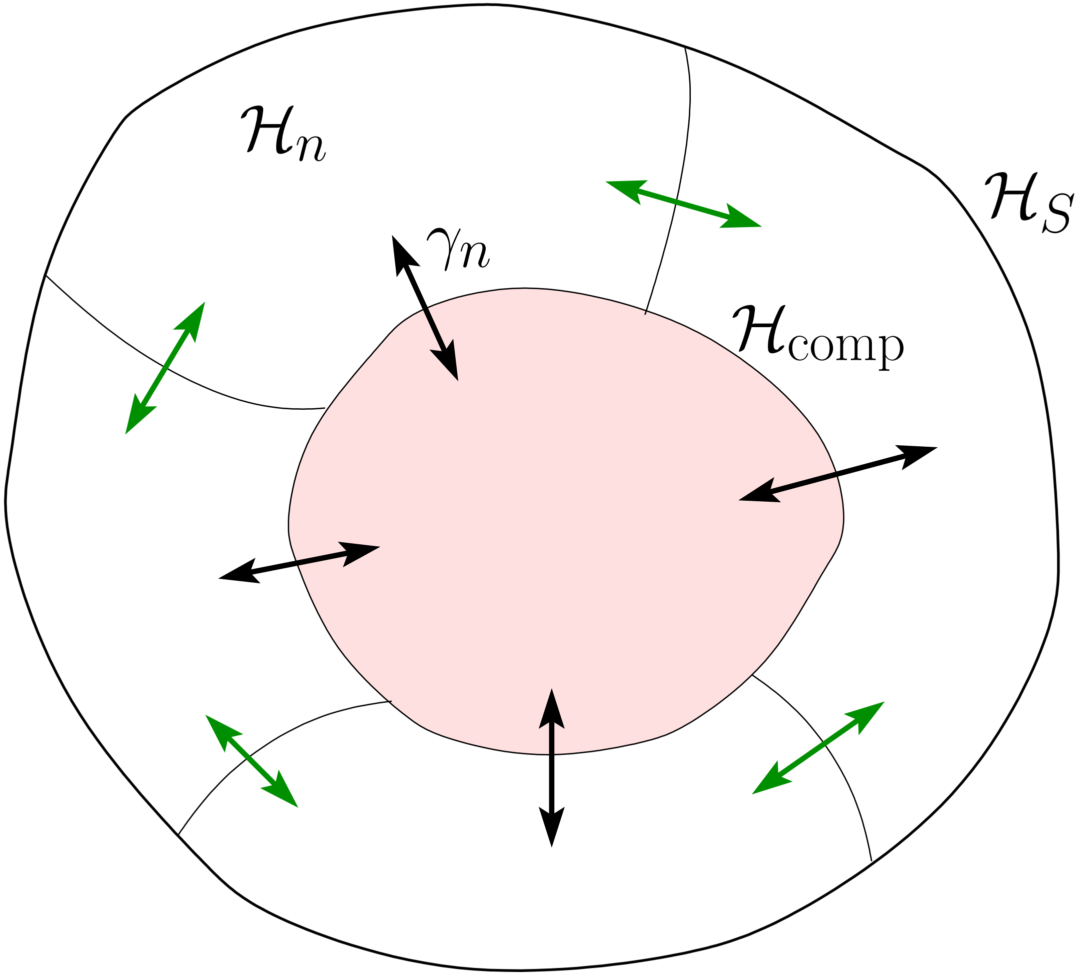
Falci, D'Arrigo, Mastellone, Paladino (2004)
Brion, Akulin, Comparat, Dumer, Harel, Kebaili, Kurizki, Mazets, Pillet (2004)
Zhang, Zhou, Yu, Guo (2004)

Original idea: *exploit symmetries* in order to halt decoherence
New perspective: **increase coupling** in order to inhibit decoherence

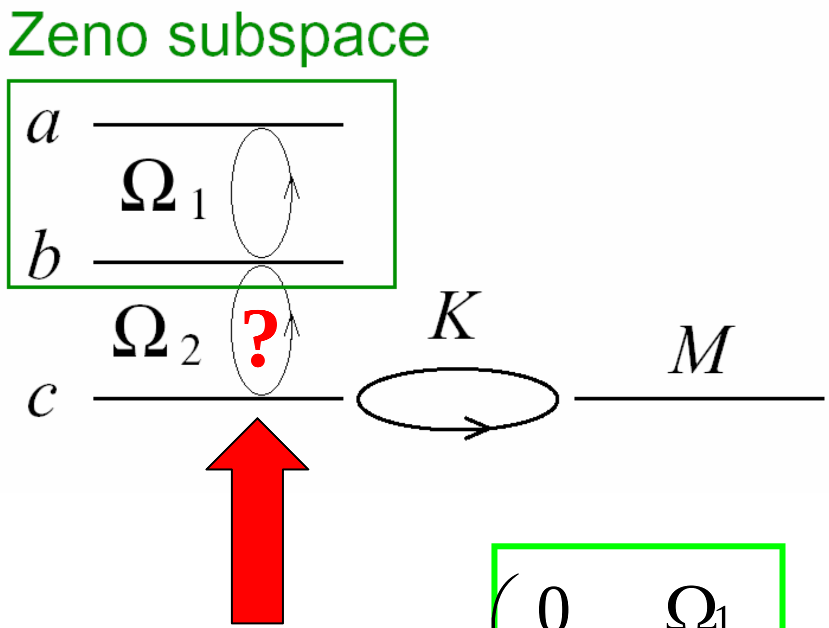
Main idea/issue



Zeno subspaces



The problem



$$K > 1/\tau_Z \cong 10^{15}\,\mathrm{s}^{-1}$$

Unstable systems

$$K > 1/\tau_Z^2\gamma \cong 10^{20}\,\mathrm{s}^{-1}$$

Unstable systems and **Inverse** Zeno

$$(\omega_b \neq 0)$$

$$H_K = H_{\mathrm{decay}} + K H_c = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & 1/\tau_Z & 0 \\ 0 & 1/\tau_Z & -i2/\tau_Z^2\gamma & K \\ 0 & 0 & K & -i2/\tau_Z^2\gamma \end{pmatrix}$$

Framework

$$H_{\text{tot}} = H_0 + H_{SB} = H_S \otimes 1_B + 1_S \otimes H_B + H_{SB}$$

decoherence

$$\mathcal{L}_{\text{tot}}\rho = -i[H_{\text{tot}}, \rho]$$

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_0 + \mathcal{L}_{\text{SB}}$$

Liouvillian

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_S \otimes \mathcal{H}_B$$

$$\mathcal{H}_S = \mathcal{H}_{\text{comp}} \oplus \mathcal{H}_{\text{orth}}$$

$$\left(e.g.: \mathcal{H}_{\text{comp}} = C^2\right)$$

Hilbert spaces

System-bath interaction (Gardiner & Zoller)

$$H_{SB} = \sum_m X_m \otimes A_m^+ + X_m^+ \otimes A_m$$

$$\mathrm{L}_S X_m = i\omega_m X_m$$

$$A_m = A(\kappa_m) = \int \kappa_m^*(\omega) a(\omega) d\omega$$

form factor

$[\kappa_m(\omega) = 0, \text{ for } \omega < 0]$

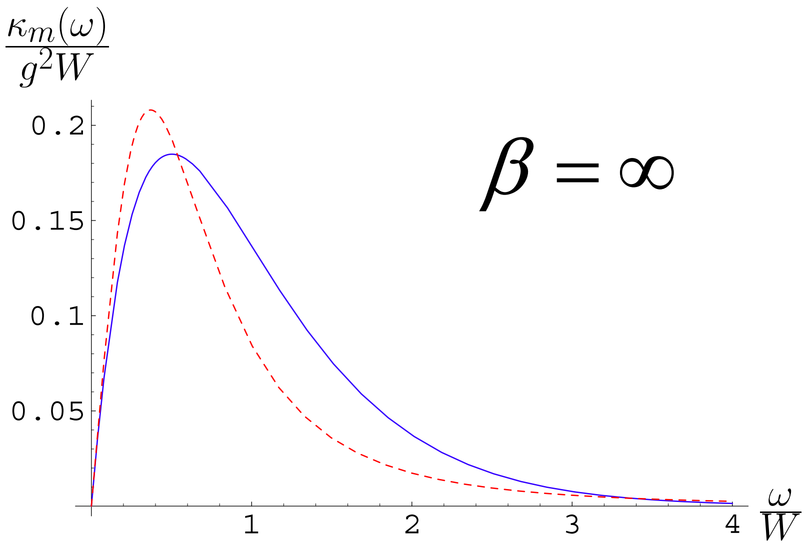
Polynomial and exponential case

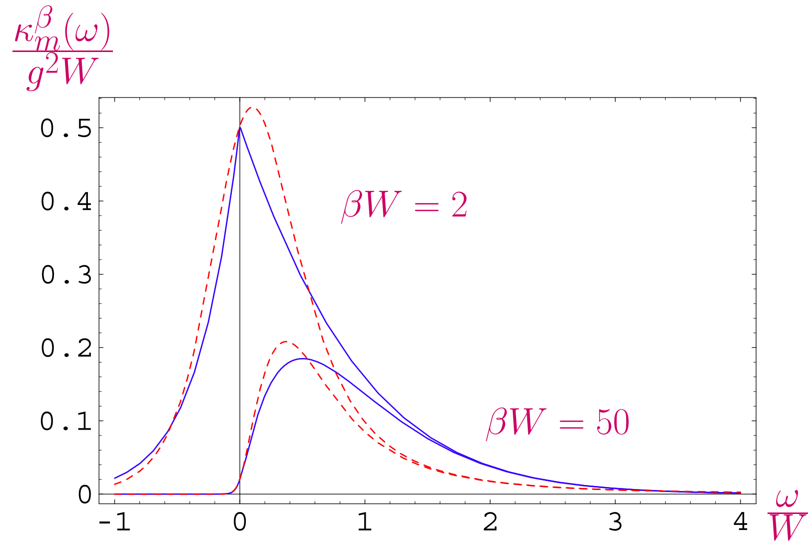
$$\kappa_m(\omega) = g^2 \omega \exp(-\omega/\Lambda)\theta(\omega) \quad \text{exponential}$$

$$\kappa_m(\omega) = g^2 \frac{\omega}{(1+(\omega/\Lambda)^2)^2}\theta(\omega) \quad \text{polynomial}$$

$$W = \tau_Z^2 \int_{-\infty}^{\infty} |\,\omega\,|\, \kappa_m(\omega) d\omega \quad \text{bandwidth}$$

Form factors





Full line: exponential; dashed line: polynomial form factor.

β inverse temperature

W bandwidth

Dynamics

$$\rho(t) = \left[\hat{P}\exp(\mathrm{L}_{\mathrm{tot}}(\tau))\,\hat{P}\right]^{t/\tau}\rho(0)$$ Zeno (measurements)

$$\rho(t) = \left[\exp(\mathrm{L}_{\mathrm{k}})\exp(\mathrm{L}_{\mathrm{tot}}(\tau))\right]^{t/\tau}\rho(0)$$ kicks

$$\rho(t) = \exp[(K\mathrm{L}_{\mathrm{c}} + \mathrm{L}_{\mathrm{tot}})t]\rho(0)$$ continuous coupling

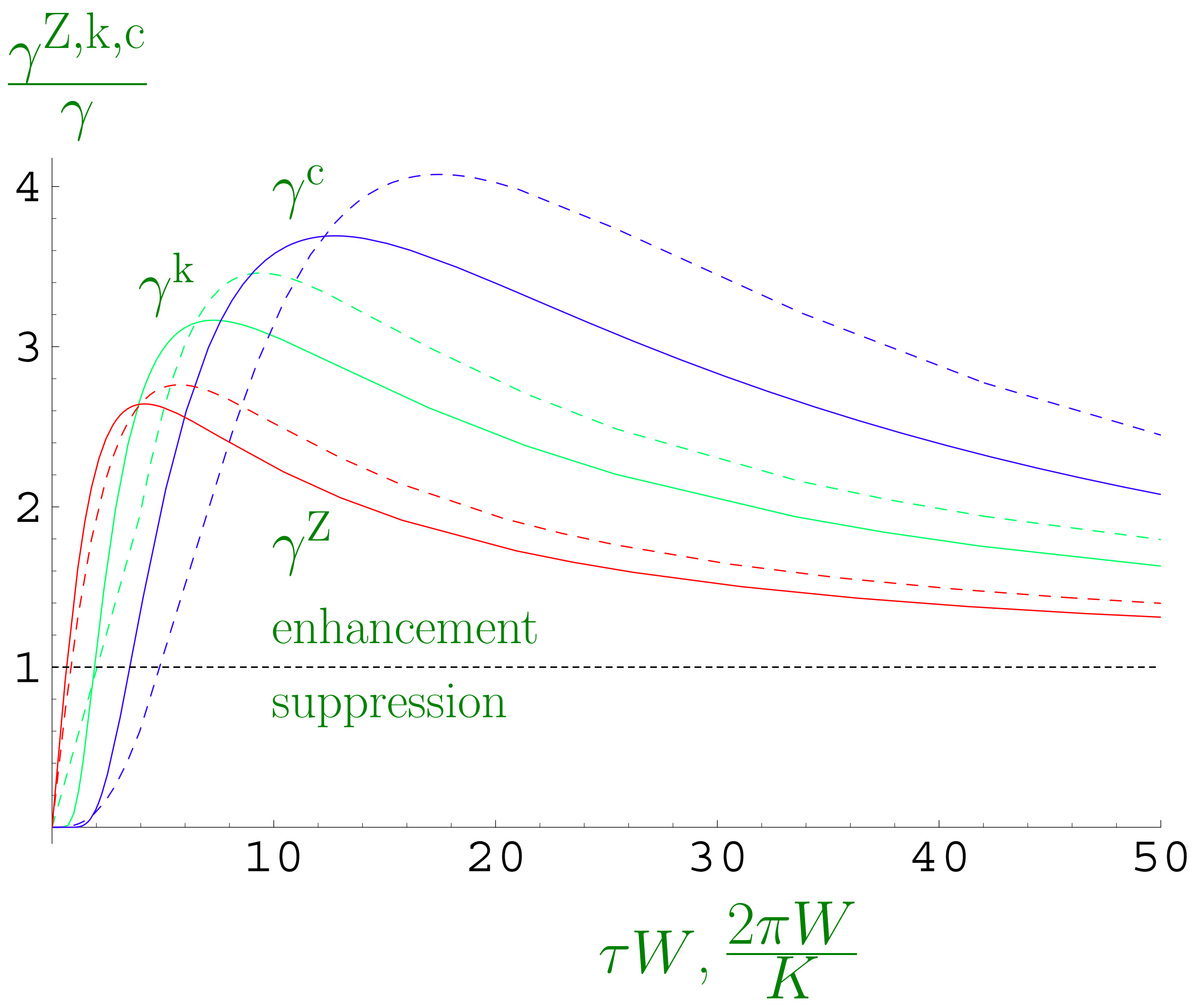
always look at the timescales γ^{-1} for dissipation

important !!

$$\tau^{*} \cong \gamma\tau_{\mathrm{Z}}^{2}$$

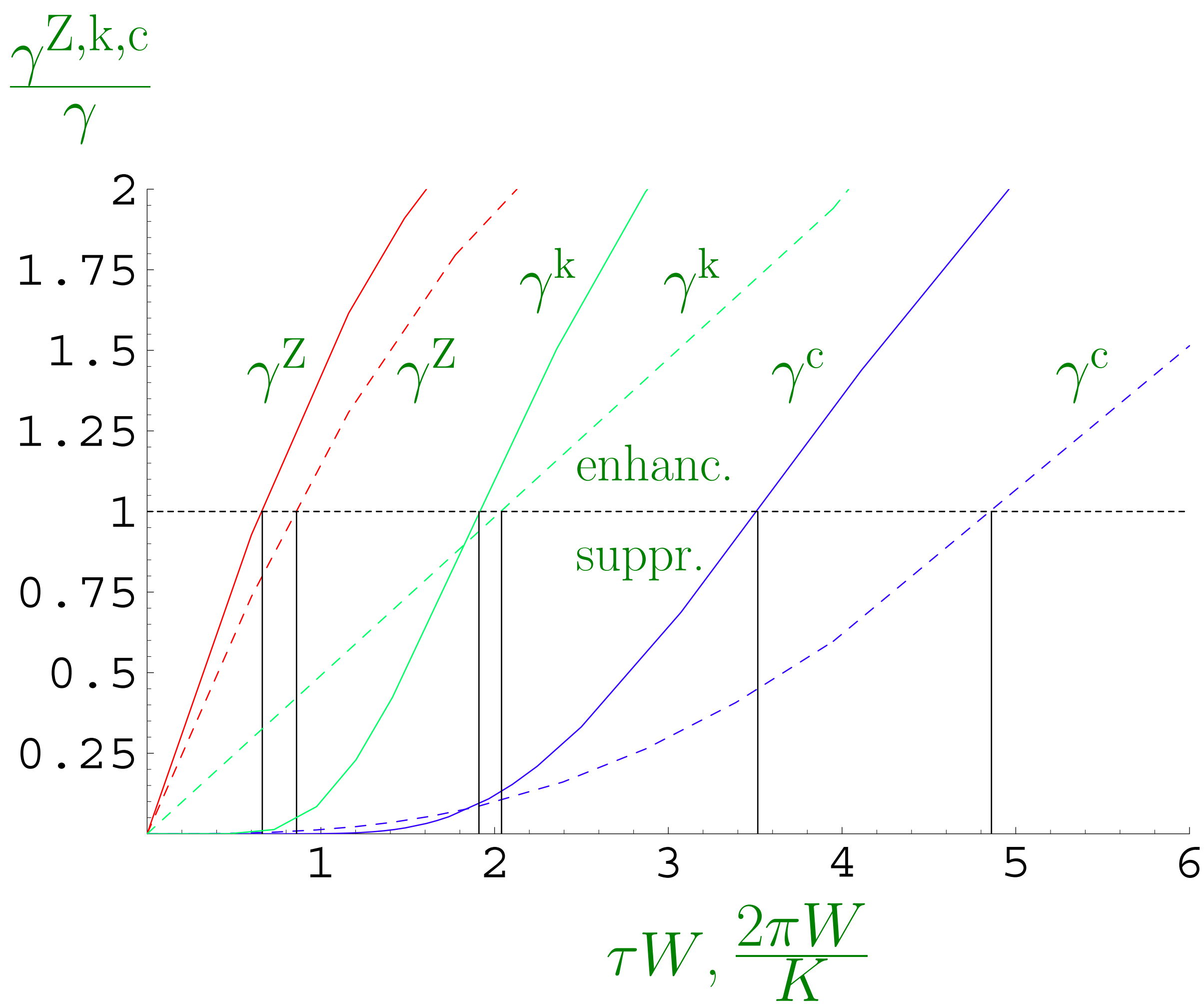
control (protection) ⟷ enhancement

Comparison



$\frac{\gamma^{\mathrm{Z,k,c}}}{\gamma}$
γ^{c}
γ^{k}
γ^{Z}
enhancement
suppression
4
3
2
1
10
20
30
40
50
$\tau W, \frac{2\pi W}{K}$

Comparison
(small times 1/*N* -- strong coupling *K*)



Remarkable differences

$$\gamma_{\text{eff}} \cong \frac{\tau}{\tau_{\text{Z}}^2} = \tau \int d\omega \kappa^{\beta}(\omega)$$

Zeno control
(non-unitary)

$$\gamma_{\text{eff}} \cong \frac{8}{\pi} \kappa^{\beta} \left(\frac{\pi}{\tau} \right)$$

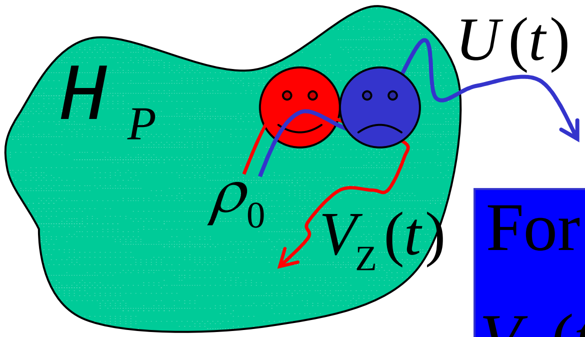
"bang bang" control
(kicks) (unitary)

$$\gamma_{\text{eff}} \cong \pi \kappa^{\beta}(K)$$

control via
continuous coupling
(unitary)

For unitary controls γ_{eff} feels the "tail" of the form factor

The quantum Zeno dynamics



Is the Zeno dynamics unitary?

(Misra and Sudarshan 1977: **semi**group)

For H bounded,

$$V_Z(t) = \lim_{N\to\infty}[PU(t/N)P]^N = P\exp(-iH_Z t)$$

unitary in $\mathcal{H}_P$, i.e. $H_Z = PHP$ self-adjoint in $\mathcal{H}_P$.

For H unbounded,

$$V_Z(t) = \lim_{N\to\infty}[PU(t/N)P]^N = P\exp(-iH_Z t)$$

PHP in general is not self-adjoint, nor even closed

Answer: under general hypotheses YES

$$H_Z = \left(H^{1/2}P\right)^\dagger \left(H^{1/2}P\right),$$

if H_Z densely defined.

Friedman 1972
Facchi, Gorini, Marmo, Pascazio, Sudarshan 2000
Facchi, Pascazio, Scardicchio, Schulman 2002
Exner, Ichinose 2003
Gustavson 2004

Free particle in D dimensions

$$H = \frac{p^2}{2M} = -\frac{\Delta^2}{2M}, \quad U(t) = \exp(-iHt) \quad \text{in } L^2(\mathbb{R}^D)$$

$\Omega \subset \mathbb{R}^D$ a compact domain, $\quad P = \chi_\Omega(x)$ spatial projection

$$\text{Zeno dynamics} \quad V_Z(t) = \lim_{N\to\infty}\left[V\left(\frac{t}{N}\right)\right]^N, \quad V(s) = PU(s)P$$

How does the particle move inside Ω? Does it leak out?

THEOREM

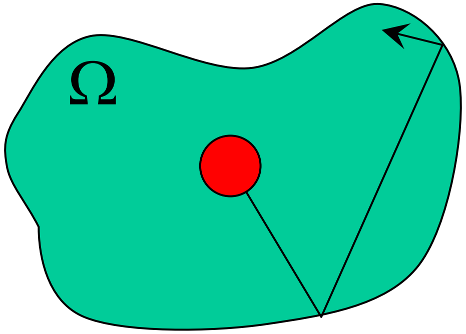
The limit $V_Z(t)$ exists and yields

$V_Z(t) = P\exp(-iH_Z t)$, a unitary group in $L^2(\Omega)$

where the Zeno Hamiltonian reads

$$H_Z = -\frac{\Delta^2}{2M}, \quad D[H_Z] = \left\{\psi \in W^2(\Omega) \mid \psi(\partial\Omega) = 0\right\}$$

Dirichlet boundary conditions



Free particle in a box with perfectly reflecting hard walls

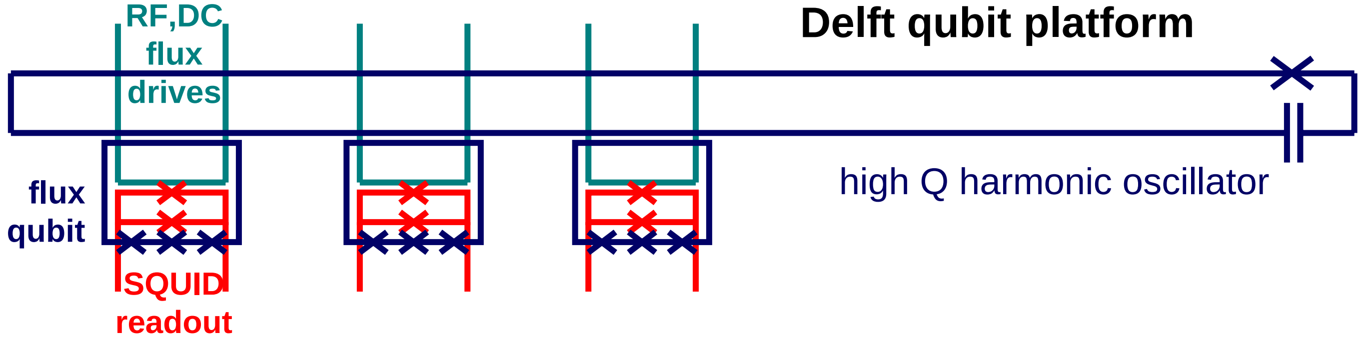
…although there is NO wall!

Facchi, Pascazio, Scardicchio, Schulman 2002
Facchi, Marmo, Pascazio, Scardicchio, Sudarshan 2003

EuroSQIP

European Superconducting Quantum Information Processor

Coordinator:
Göran Wendin, Chalmers



EuroSQIP partners

1. Chalmers (Chalmers University of Technology, Sweden) (CO)
2. CEA (SPEC, CEA-Saclay, France)
3. TUD (Delft University of Technology, The Netherlands)
4. PTB (Physikalisch-Technische Bundesanstalt, Germany)
5. CNRS (Grenoble & Toulouse, France)
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8. IPHT (Institut für Physikalische Hochtechnologie, Jena, Germany)

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13. LMU (Ludwig-Maximilians-Universität, München)

14. LANDAU (Landau Institute for Theoretical Physics, Moscow)

Work Packages

WP.1 Quantum processors

WP.2 Readout of quantum information

WP.3 Interfaces and storage of quantum information

WP.4 Implementation of algorithms and protocols

WP.5 Systems integration and demonstration

WP.6 Training and dissemination

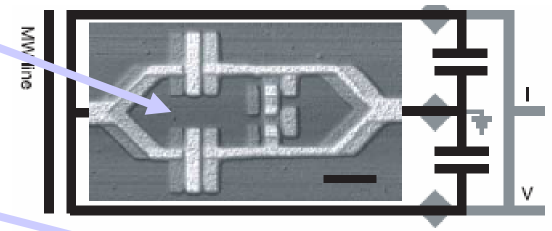
WP.7 Review and assessment

WP.8 Project management

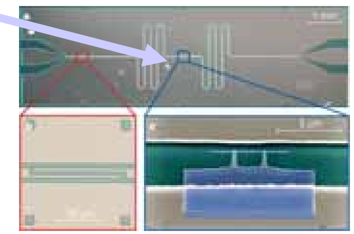
Coupling between JJ qubits and photons

TU Delft: Photons in Josephson resonators
I. Chiorecu et al.. Nature 431, 159 (2004)

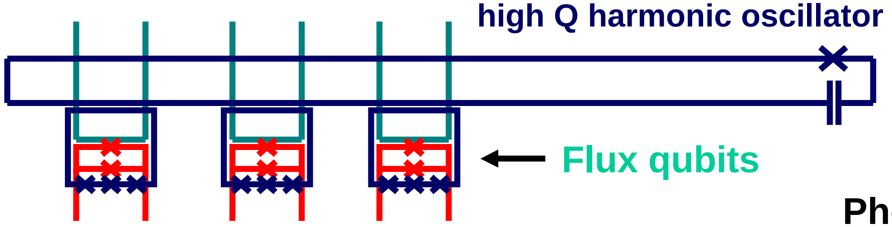
Sideband physics

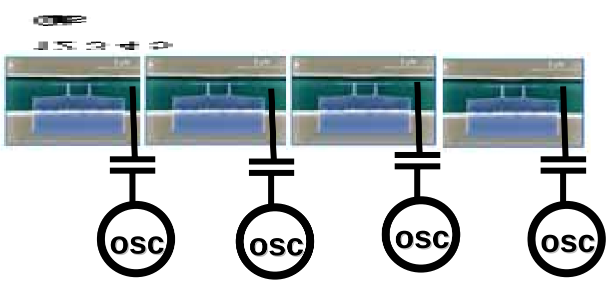


Yale: Photons in stripline resonators
A. Wallraff et al. Nature 431, 162 (2004)



Goal: Hardware integration with microwave/optical quantum communication





Photons typically coupled through guiding circuitry
Use microwave transmission lines / cascaded quantum oscillator systems for transferring microwave photons
Use interface to Ion traps for accessing the optical world

Use other frequency conversion techniques known from optics

Superconducting qubits can be entangled with microwave photons in transmission lines through high-Q cavities in all designs