Neutron optical experiments exploring foundamental quantum-phenomena

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1. Neutron interferometer/polarimeter

2. Recent neutron optical experiments

2-1 Quantum contextuality
- Violation of a Bell-like inequality
- Kochen-Specker-like paradox

2-2 Quantum state tomography

2-3 Geometric phase
- Non-cyclic spatial geometric phase
- Geometric phase for mixed state

3. Summary

Neutron interferometry

Neutrons

$m = 1.67 \times 10^{-27}$ kg

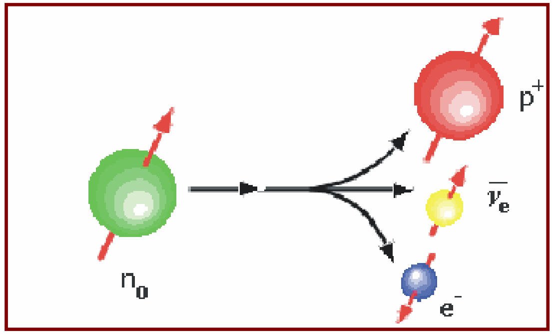
$s = \frac{1}{2}\hbar$

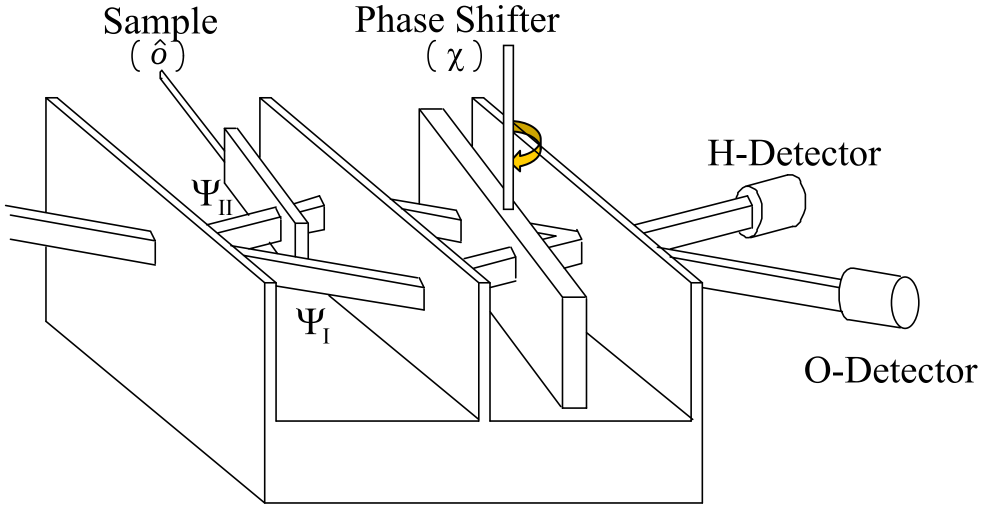
$\mu = -9.66 \times 10^{-27}$ J/T

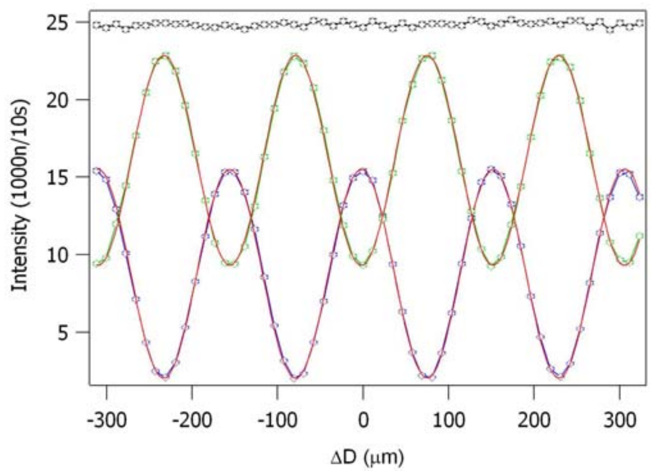
$\tau = 887$ s

$R = 0.7$ fm

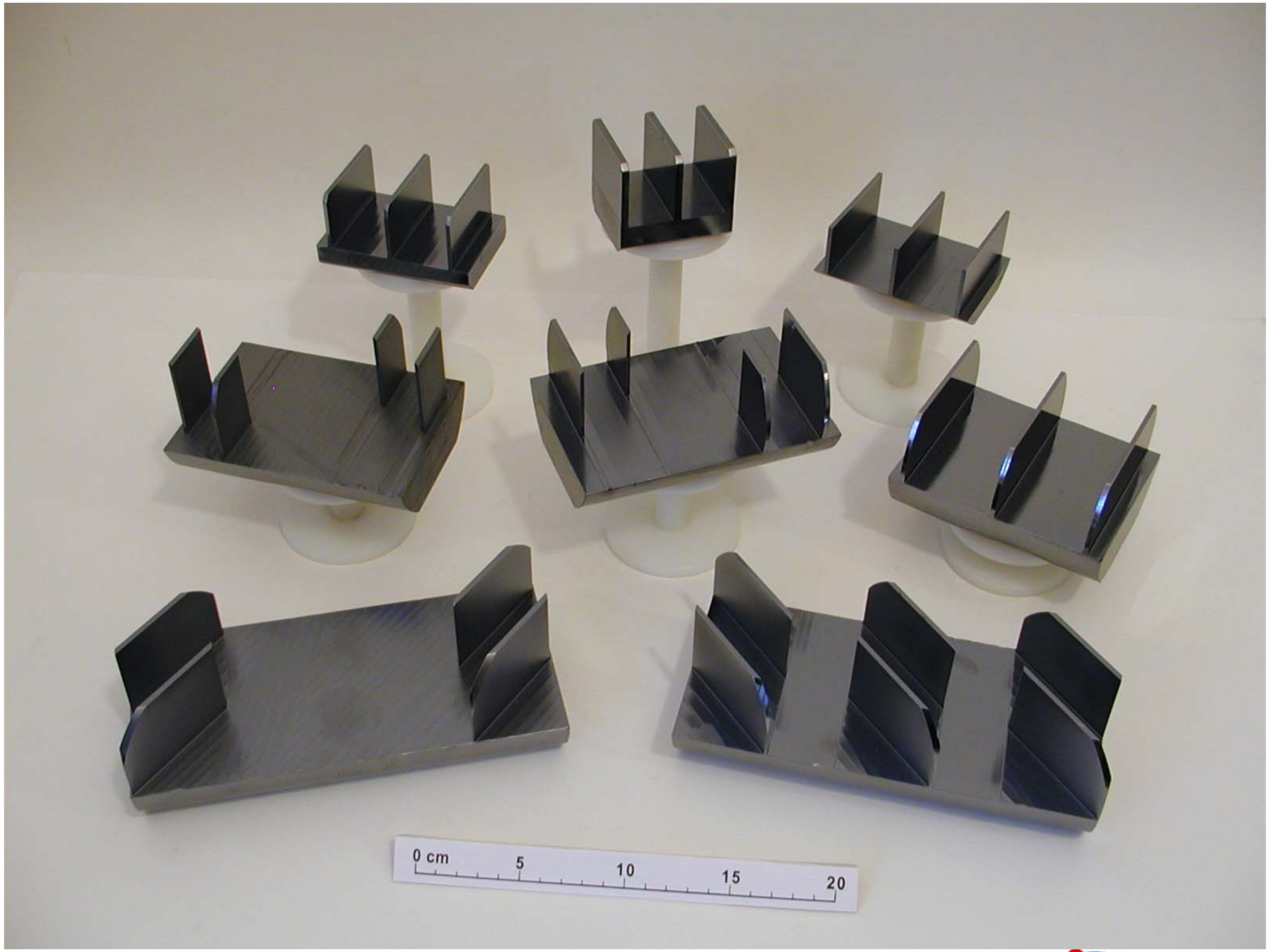
u–d–d quark structure



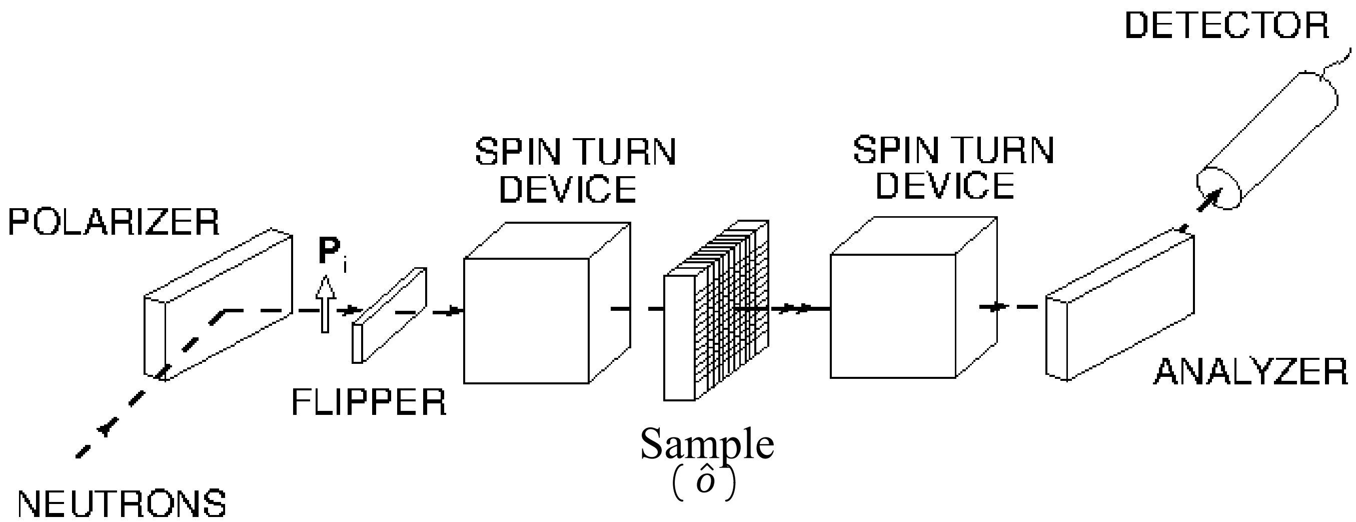




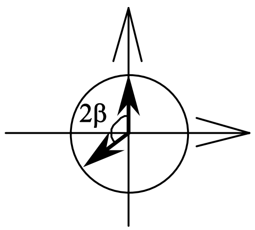
Neutron interferometers



Larmor interferometry – neutron polarimeter



$$\hat{O}\,|\pm Y\rangle = -e^{\pm i\beta}\cdot|\pm Y\rangle \;\Rightarrow\Rightarrow$$



$$\begin{aligned}
&\hat{O}|+Z\rangle \\
&= \hat{O}\left\{\frac{1}{\sqrt{2}}\left(|+Y\rangle + |-Y\rangle\right)\right\} \\
&= \frac{1}{\sqrt{2}}\left(e^{i\beta}\cdot|+Y\rangle + e^{-i\beta}\cdot|-Y\rangle\right) \\
&= \frac{e^{i\beta}}{\sqrt{2}}\left(|+Y\rangle + e^{-2i\beta}\cdot|-Y\rangle\right)
\end{aligned}$$

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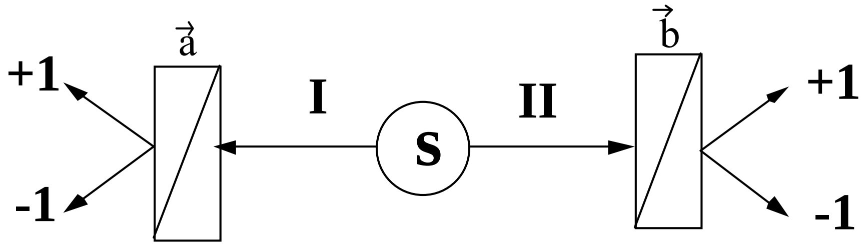
From two-particle to two-space entanglement



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

⇒⇒⇒ Entanglement between *Two-Particles*

2-Particle Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

I, II represent 2-Particles

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_s \otimes |\,I\rangle_p + |\downarrow\rangle_s \otimes |\,II\rangle_p\}$$

s, p represent 2-Spaces, e.g., spin & path

(Non-)Contextuality ==>> Bell-like inequality

(In)Dependent Results for commuting Observables

Bell-like inequality with E'(α,χ)

$$-2 \le S' \le 2$$

with $S' = E'(\alpha_1,\chi_1) - E'(\alpha_1,\chi_2) + E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$

where
$$E'(\alpha,\chi) = \frac{N'_{++}(\alpha,\chi) + N'_{--}(\alpha,\chi) - N'_{+-}(\alpha,\chi) - N'_{-+}(\alpha,\chi)}{N'_{++}(\alpha,\chi) + N'_{--}(\alpha,\chi) + N'_{+-}(\alpha,\chi) + N'_{-+}(\alpha,\chi)}$$
$$= \frac{N'_{++}(\alpha,\chi) + N'_{++}(\alpha+\pi,\chi+\pi) - N'_{++}(\alpha,\chi+\pi) - N'_{++}(\alpha+\pi,\chi)}{N'_{++}(\alpha,\chi) + N'_{++}(\alpha+\pi,\chi+\pi) + N'_{++}(\alpha,\chi+\pi) + N'_{++}(\alpha+\pi,\chi)}$$

$$N'_{\pm\pm}(\alpha,\chi) = \langle \Psi | \hat{P}^{s}_{\alpha;\pm 1} \cdot \hat{P}^{p}_{\chi;\pm 1} | \Psi \rangle$$

Prediction by quantum theroy

$N'^{qm}_{++}(\alpha,\chi) = \frac{1}{2}\{1+\cos(\alpha+\chi)\}$ as well as $E'(\alpha,\chi) = \cos(\alpha+\chi)$,

then, $S' = 2\sqrt{2} = 2.82 > 2$ for $\begin{cases} \alpha_1 = \pi/2,\ \alpha_2 = 0 \\ \chi_1 = -\pi/4,\ \chi_2 = \pi/4 \end{cases}$

Remark: contrast!
$\bar{C} > 71\%$

Contextuality in quantum mechanics

Non-contextuality:

Independent results: $v[\hat{\mathrm{A}}^{\mathrm{s}}\,\hat{\mathrm{B}}^{\mathrm{p}}] = v[\hat{\mathrm{A}}^{\mathrm{s}}]\cdot v[\hat{\mathrm{B}}^{\mathrm{p}}]$

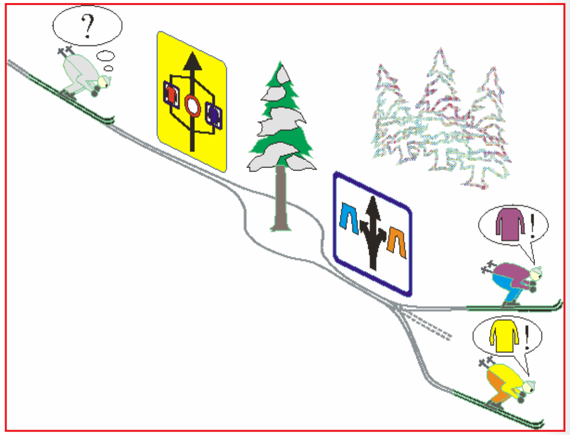
for measurements of the commuting observables, $[\hat{\mathrm{A}}^{\mathrm{s}}, \hat{\mathrm{B}}^{\mathrm{p}}] = 0$.

➔➔Non-locality is one aspect of contextuality

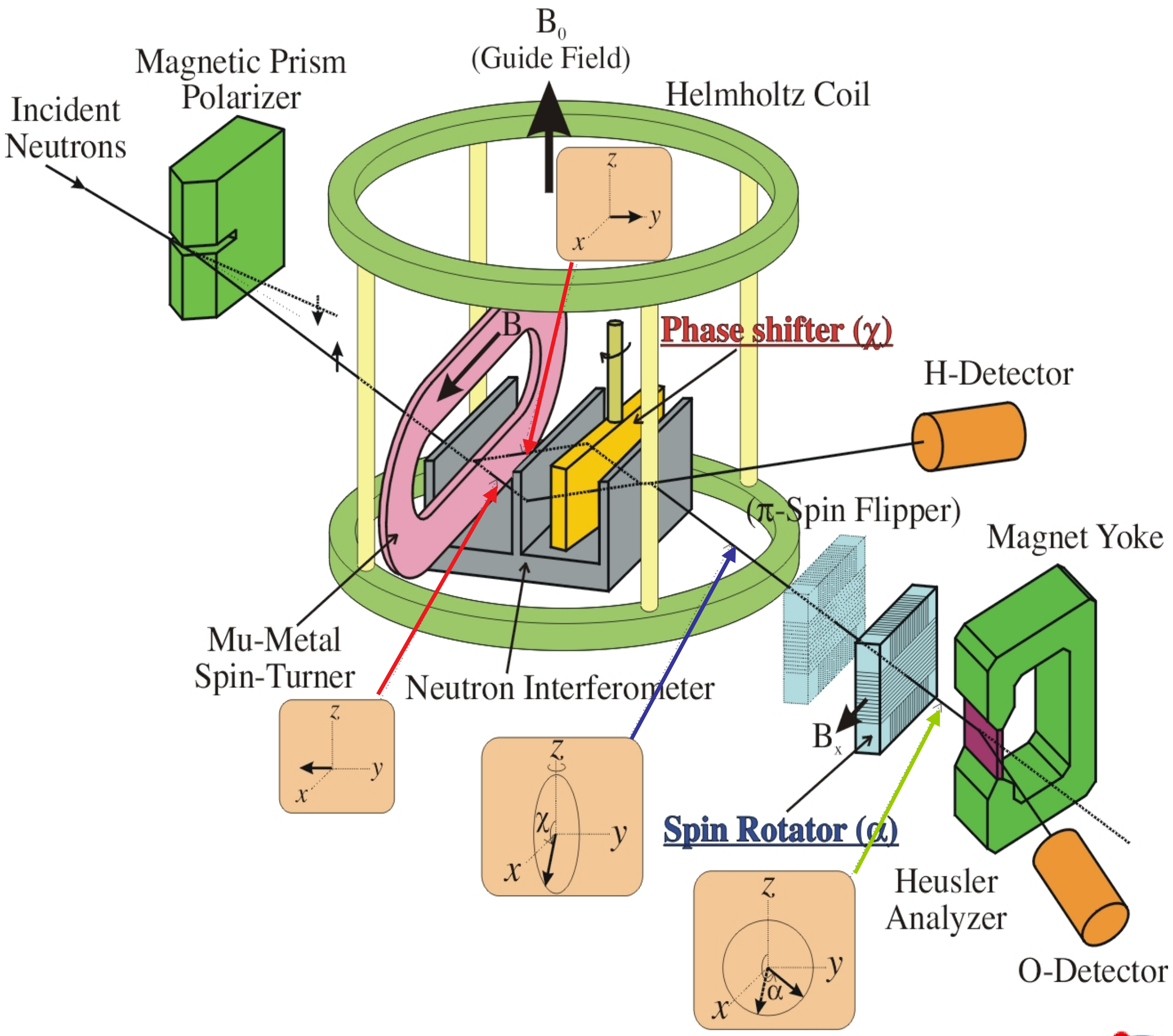
$([\hat{P}^{\mathrm{I}(\mathbf{r}_{\mathrm{I}})}, \hat{P}^{\mathrm{II}(\mathbf{r}_{\mathrm{II}})}] = 0$, since $\mathbf{r}_{\mathrm{I}} \neq \mathbf{r}_{\mathrm{II}})$

In quantum muchanics:

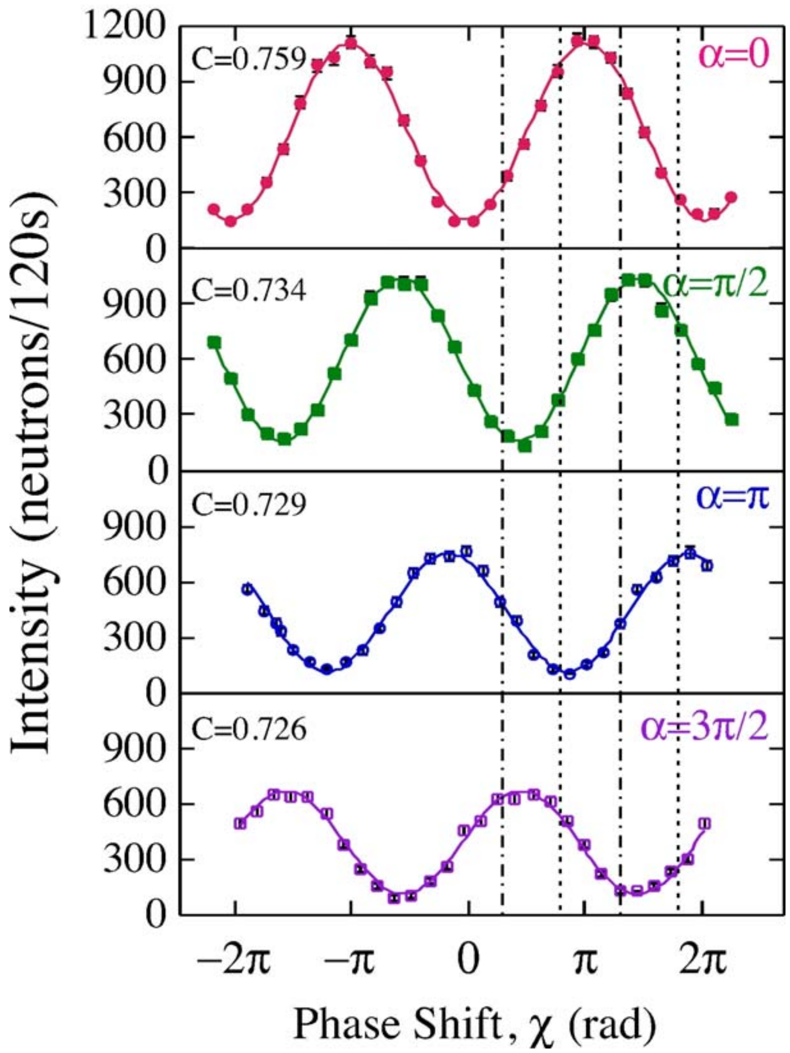
$\left.\begin{matrix} \textit{Non–local} \\ \textit{Contextual} \end{matrix}\right\}$ *correlations* are expected



Schematic view of the experiment



Violation of a Bell-like inequality



$$E'(\alpha,\chi) = \frac{N'_{++}(\alpha,\chi) + N'_{++}(\alpha+\pi,\chi+\pi) - N'_{++}(\alpha,\chi+\pi) - N'_{++}(\alpha+\pi,\chi)}{N'_{++}(\alpha,\chi) + N'_{++}(\alpha+\pi,\chi+\pi) + N'_{++}(\alpha,\chi+\pi) + N'_{++}(\alpha+\pi,\chi)}$$

where $N'_{++}(\alpha,\chi) = \langle\Psi|\hat{P}^{s}_{(\alpha)}\cdot\hat{P}^{p}_{(\chi)}|\Psi\rangle$

$$\begin{cases} E'(\alpha_1,\chi_1) = 0.542 \pm 0.007 \\ E'(\alpha_1,\chi_2) = 0.488 \pm 0.012 \\ E'(\alpha_2,\chi_1) = -0.538 \pm 0.006 \\ E'(\alpha_2,\chi_2) = 0.483 \pm 0.012 \end{cases} \quad \text{where} \begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0.50\pi \\ \chi_1 = 0.79\pi \\ \chi_2 = 1.29\pi \end{cases}$$

===>>> $S' \equiv E'(\alpha_1,\chi_1) + E'(\alpha_1,\chi_2) - E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$

$= \mathbf{2.051 \pm 0.019 \quad > 2}$

Cf. Max. violaion: S'=2.81>2

Y. Hasegawa et al., Nature Vol. 425, Sept. 4, 2003

Non-locality & contextuality

	Non – Locality Correlated results for $\hat{A}(r_1)$ & $\hat{A}(r_2)$ $(r_1 \neq r_2)$	**Contextuality** Correlated results for $\hat{A}$ & $\hat{B}$ $([\hat{A},\hat{B}]=0)$
Statistical Violation	**Bell State** $\lvert\Psi\rangle = \frac{1}{\sqrt{2}}\{\lvert a\rangle\otimes\lvert b\rangle + \lvert\bar{a}\rangle\otimes\lvert\bar{b}\rangle\}$ **Bell's Inequality** $\lvert S\rvert \leq 2$, where $S \equiv E(a_1,b_1)+E(a_1,b_2)-E(a_2,b_1)+E(a_2,b_2)$ (1964)	(2001)
Contradiction	**Greenberger-Horne-Zeilinger (GHZ) State** $\lvert\Psi\rangle = \frac{1}{\sqrt{2}}\{\lvert H\rangle_1\lvert H\rangle_2\lvert H\rangle_3 + \lvert V\rangle_1\lvert V\rangle_2\lvert V\rangle_3\}$ $= \frac{1}{2}\{\lvert R\rangle_1\lvert L\rangle_2\lvert H'\rangle_3 + \lvert L\rangle_1\lvert R\rangle_2\lvert H'\rangle_3 + \lvert R\rangle_1\lvert R\rangle_2\lvert V'\rangle_3 + \lvert L\rangle_1\lvert L\rangle_2\lvert V'\rangle_3\}$ (1990)	**Kochen-Specker (KS) Theorem** $v[O] = \pm 1$ $\begin{cases} v[A]\cdot v[B] = +1 \\ v[B]\cdot v[C] = +1 \\ v[C]\cdot v[A] = -1 \end{cases}$ (1967)

Theory (1)

Bell-state for spin & path entanglement

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle_s \otimes |\,\mathrm{I}\rangle_p + |\downarrow\rangle_s \otimes |\,\mathrm{II}\rangle_p\right\}$$

Rem.: $\begin{cases} \hat{X}_1 \hat{X}_2 |\Psi\rangle = |\Psi\rangle \\ \hat{Y}_1 \hat{Y}_2 |\Psi\rangle = |\Psi\rangle \end{cases}$

$$\hat{X}_1 \hat{Y}_2 |\Psi\rangle = -\hat{Y}_1 \hat{X}_2 |\Psi\rangle$$

Pauli-type observables for spin and path

$$\begin{cases} \hat{X}_1 = \hat{\sigma}_x^s = \hat{A}^s(0) = (+1)\cdot\hat{P}^s_{(|+x\rangle)} + (-1)\cdot\hat{P}^s_{(|-x\rangle)} \\ \hat{X}_2 = \hat{\sigma}_x^p = \hat{B}^p(0) = (+1)\cdot\hat{P}^p_{(|+x\rangle)} + (-1)\cdot\hat{P}^p_{(|-x\rangle)} \end{cases}$$

$$\begin{cases} \hat{Y}_1 = \hat{\sigma}_y^s = \hat{A}^s\left(\frac{\pi}{2}\right) = (+1)\cdot\hat{P}^s_{(|+y\rangle)} + (-1)\cdot\hat{P}^s_{(|-y\rangle)} \\ \hat{Y}_2 = \hat{\sigma}_y^p = \hat{B}^p\left(\frac{\pi}{2}\right) = (+1)\cdot\hat{P}^p_{(|+y\rangle)} + (-1)\cdot\hat{P}^p_{(|-y\rangle)} \end{cases}$$

Here, $[\hat{X}_1, \hat{X}_2] = 0,\ [\hat{Y}_1, \hat{Y}_2] = 0$

$[\hat{X}_1, \hat{Y}_2] = 0,\ [\hat{Y}_1, \hat{X}_2] = 0$

Product Observables

$$\hat{C} \equiv \hat{I} + \hat{X}_1 \hat{X}_2 + \hat{Y}_1 \hat{Y}_2 - \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2$$

The following equality holds for these parameters

$$C = 1 + v[\hat{X}_1 \hat{X}_2] + v[\hat{Y}_1 \hat{Y}_2] - v[\hat{X}_1 \hat{Y}_2]\cdot v[\hat{Y}_1 \hat{X}_2]$$

where $v[\hat{X}_j] = \pm 1$ & $v[\hat{Y}_j] = \pm 1$

Theory (2)

Non-Contextuality Hidden Variable (NCHV) theory leads to

$$C_{NCHV} = 1 + v[\hat{X}_1]\cdot v[\hat{X}_2] + v[\hat{Y}_1]\cdot v[\hat{Y}_2] - v[\hat{X}_1]\cdot v[\hat{Y}_2]\cdot v[\hat{Y}_1]\cdot v[\hat{X}_2]$$
$$= v[\hat{X}_1]\cdot v[\hat{X}_2]\cdot(1 - v[\hat{Y}_1]\cdot v[\hat{Y}_2]) + (1 + v[\hat{Y}_1]\cdot v[\hat{Y}_2])$$
$$= \pm 2$$

Then,

$$|C_{NCHV}| = \left| 1 + \langle \hat{X}_1 \hat{X}_2 \rangle + \langle \hat{Y}_1 \hat{Y}_2 \rangle - \langle \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2 \rangle \right|$$
$$\leq 2 \quad \Leftrightarrow \mathbf{C_{QM} = 4}$$

Independent results for the commuting observables,

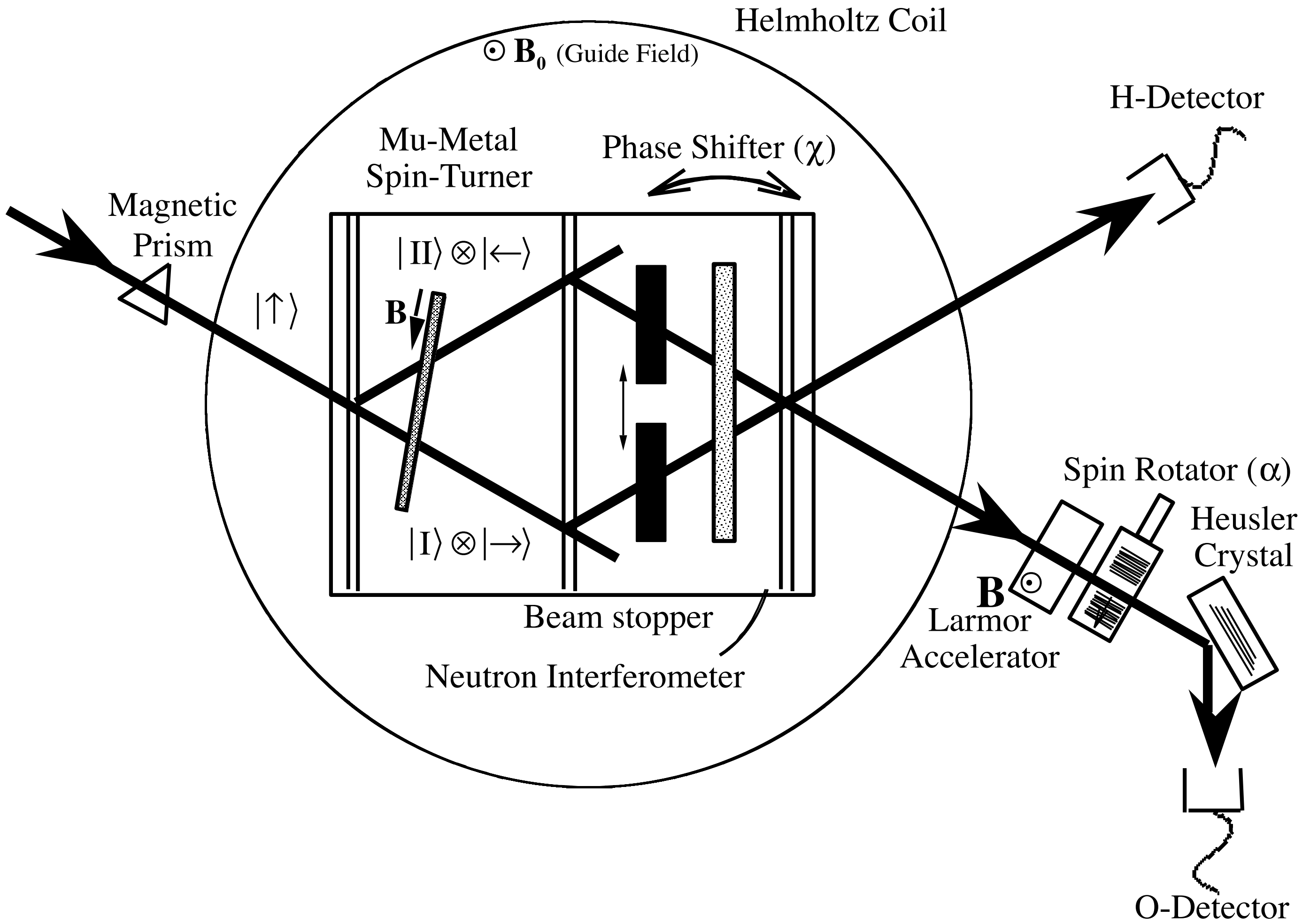
$$[\hat{X}_1, \hat{X}_2] = 0,\ [\hat{Y}_1, \hat{Y}_2] = 0,\ [\hat{X}_1, \hat{Y}_2] = 0,\ [\hat{Y}_1, \hat{X}_2] = 0$$

Quantum mechanics predicts

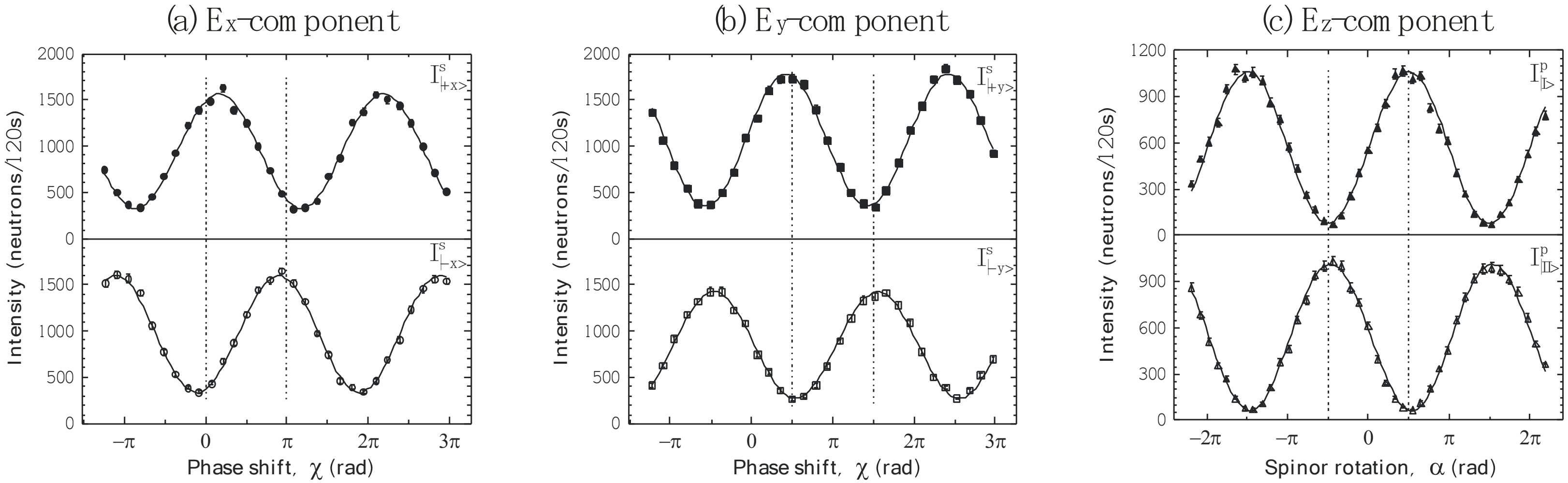
$$C_{QM} = \langle \Psi | \left(\hat{I} + \hat{X}_1 \hat{X}_2 + \hat{Y}_1 \hat{Y}_2 - \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2 \right) | \Psi \rangle$$
$$= 1 + \langle \Psi | \hat{X}_1 \hat{X}_2 | \Psi \rangle + \langle \Psi | \hat{Y}_1 \hat{Y}_2 | \Psi \rangle - \langle \Psi | \hat{X}_1 \hat{Y}_1 \cdot \hat{Y}_2 \hat{X}_2 | \Psi \rangle$$
$$= 1 + \langle \Psi | \hat{X}_1 \hat{X}_2 | \Psi \rangle + \langle \Psi | \hat{Y}_1 \hat{Y}_2 | \Psi \rangle + \langle \Psi | \hat{Z}_1 \hat{Z}_2 | \Psi \rangle$$
$$= 4$$

ATI JST FWF

Experimental setup



Results



$$C_x = \langle \Psi | \hat{X}_1 \hat{X}_2 | \Psi \rangle$$

$$= \langle \Psi | \hat{P}^s_{(|+x\rangle)} \hat{P}^p_{(\chi=0)} | \Psi \rangle + \langle \Psi | \hat{P}^s_{(|-x\rangle)} \hat{P}^p_{(\chi=\pi)} | \Psi \rangle - \langle \Psi | \hat{P}^s_{(|+x\rangle)} \hat{P}^p_{(\chi=\pi)} | \Psi \rangle - \langle \Psi | \hat{P}^s_{(|-x\rangle)} \hat{P}^p_{(v=0)} | \Psi \rangle$$

$$= 0.610 \pm 0.008$$

$$C_y = \langle \Psi | \hat{Y}_1 \hat{Y}_2 | \Psi \rangle$$

$$= \langle \Psi | \hat{P}^s_{(|+y\rangle)} \hat{P}^p_{(\chi=\pi/2)} | \Psi \rangle + \langle \Psi | \hat{P}^s_{(|-y\rangle)} \hat{P}^p_{(\chi=3\pi/2)} | \Psi \rangle - \langle \Psi | \hat{P}^s_{(|+y\rangle)} \hat{P}^p_{(\chi=3\pi/2)} | \Psi \rangle - \langle \Psi | \hat{P}^s_{(|-y\rangle)} \hat{P}^p_{(\chi=\pi/2)} | \Psi \rangle$$

$$= 0.667 \pm 0.008$$

$$C_z = \langle \Psi | \hat{Z}_1 \hat{Z}_2 | \Psi \rangle$$

$$= \langle \Psi | \hat{P}^s_{(\chi=+\pi/2)} \hat{P}^p_{(|\mathrm{I}\rangle)} | \Psi \rangle + \langle \Psi | \hat{P}^s_{(\chi=-\pi/2)} \hat{P}^p_{(|\mathrm{II}\rangle)} | \Psi \rangle - \langle \Psi | \hat{P}^s_{(\chi=-\pi/2)} \hat{P}^p_{(|\mathrm{II}\rangle)} | \Psi \rangle - \langle \Psi | \hat{P}^s_{(\chi=+\pi/2)} \hat{P}^p_{(|\mathrm{I}\rangle)} | \Psi \rangle$$

$$= 0.861 \pm 0.010$$

$$\Rightarrow\Rightarrow\Rightarrow \; C_{measured} = 1 + C_x + C_y + C_z = 3.138 \pm 0.015 \quad > 2$$

Contradictions in violation?

$E_x = 0.610$

$E_y = 0.667$

→ → $E_{expected} = +0.407$

(63%)

$E_{measured} = -0.861$

Ideally

$E_{exp} = +1$

perfect contradiction

$E_{msr} = -1$

Classical and quantum inteterference
21. Oct. 2005, Palacký University, Olomouc, Czech Republic

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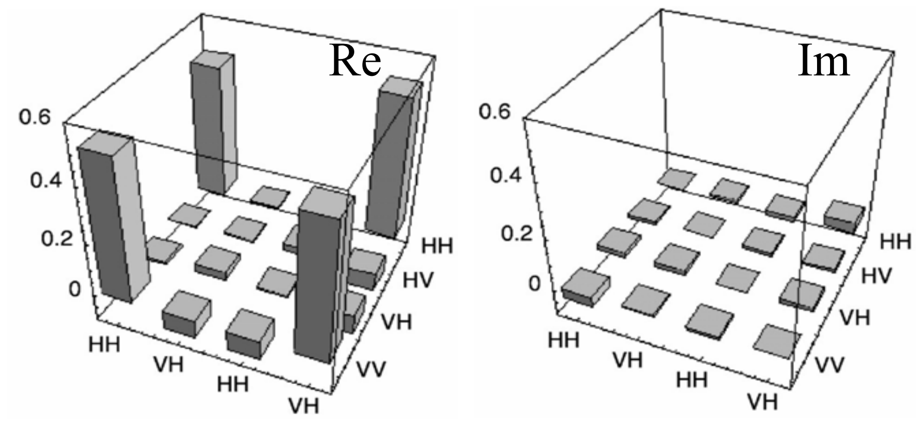
Quantum state tomography of entangled 2-qubits

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{H}\rangle + |\mathrm{V}\rangle|\mathrm{V}\rangle\right) \rightarrow \rho = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$



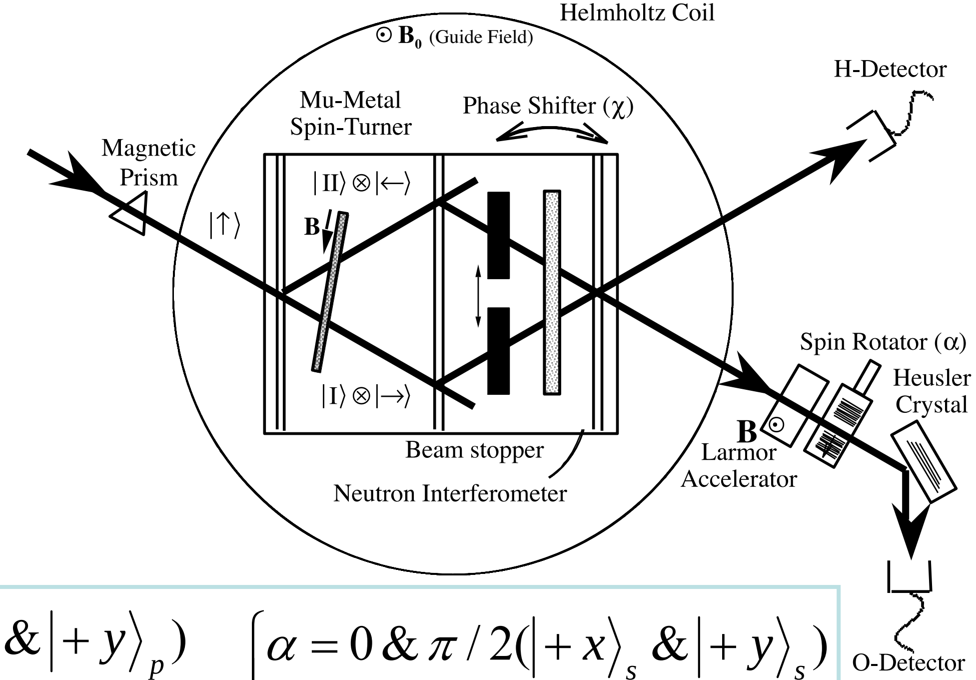
$$\hat{\rho} = \begin{pmatrix} 0.4872 & -0.0042+i0.0114 & -0.0098-i0.0178 & 0.5192+i0.0380 \\ -0.0042-i0.0114 & 0.0045 & 0.0271-i0.0146 & -0.0648-i0.0076 \\ -0.0098+i0.0178 & 0.0271+i0.0146 & 0.0062 & -0.0695+i0.0134 \\ 0.5192-i0.0380 & -0.0648+i0.0076 & -0.0695-i0.0134 & 0.5020 \end{pmatrix}$$

ν	Mode 1	Mode 2	h_1	q_1	h_2	q_2
1	$\|H\rangle$	$\|H\rangle$	45°	0	45°	0
2	$\|H\rangle$	$\|V\rangle$	45°	0	0	0
3	$\|V\rangle$	$\|V\rangle$	0	0	0	0
4	$\|V\rangle$	$\|H\rangle$	0	0	45°	0
5	$\|R\rangle$	$\|H\rangle$	22.5°	0	45°	0
6	$\|R\rangle$	$\|V\rangle$	22.5°	0	0	0
7	$\|D\rangle$	$\|V\rangle$	22.5°	45°	0	0
8	$\|D\rangle$	$\|H\rangle$	22.5°	45°	45°	0
9	$\|D\rangle$	$\|R\rangle$	22.5°	45°	22.5°	0
10	$\|D\rangle$	$\|D\rangle$	22.5°	45°	22.5°	45°
11	$\|R\rangle$	$\|D\rangle$	22.5°	0	22.5°	45°
12	$\|H\rangle$	$\|D\rangle$	45°	0	22.5°	45°
13	$\|V\rangle$	$\|D\rangle$	0	0	22.5°	45°
14	$\|V\rangle$	$\|L\rangle$	0	0	22.5°	90°
15	$\|H\rangle$	$\|L\rangle$	45°	0	22.5°	90°
16	$\|R\rangle$	$\|L\rangle$	22.5°	0	22.5°	90°

D.F. James et al., Phys. Rev. A **64** (2001) 052312.

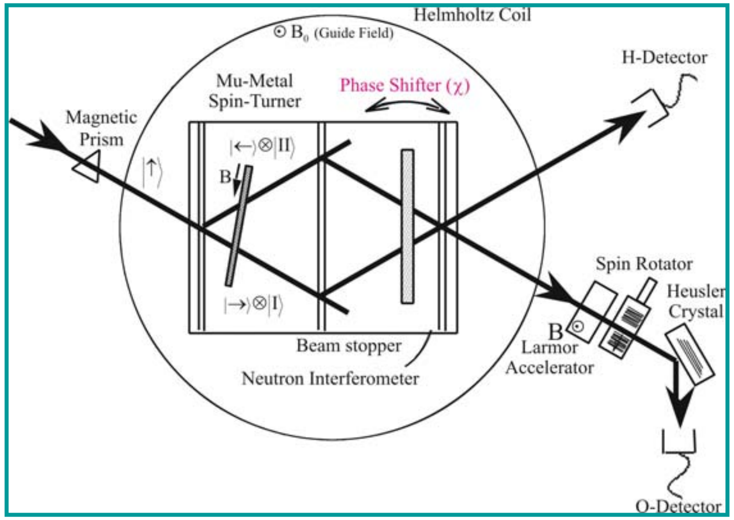
Experimental setup

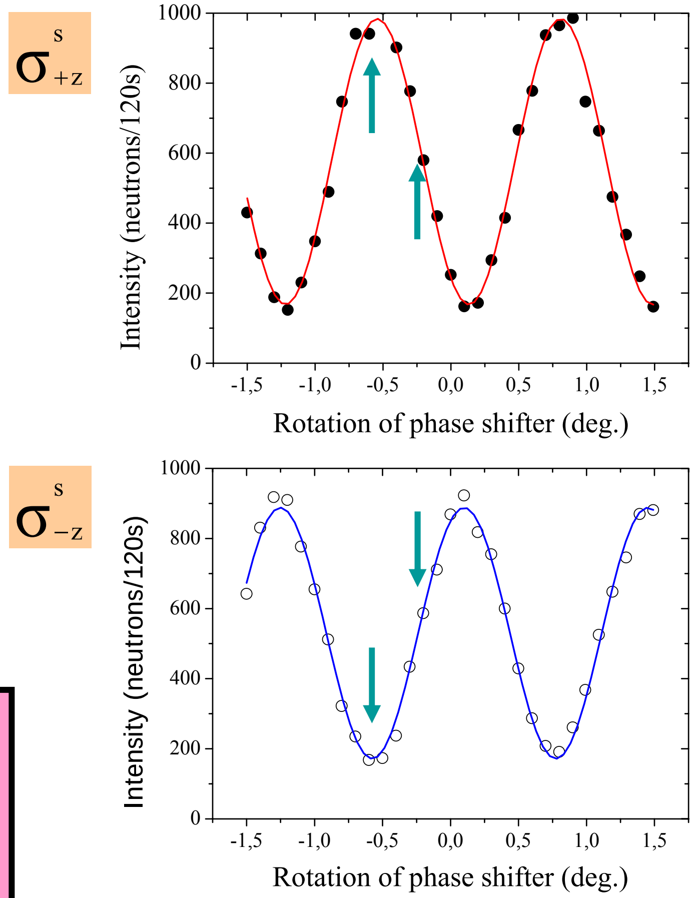
$$\left\{\sigma^{p}_{+x},\sigma^{p}_{+y},\sigma^{p}_{\pm z}\right\}\otimes\left\{\sigma^{s}_{+x},\sigma^{s}_{+y},\sigma^{s}_{\pm z}\right\}$$:16-variables to be measured!



$$\begin{cases}\chi = 0\ \&\ \pi/2\left(\left|+x\right\rangle_p\ \&\left|+y\right\rangle_p\right)\\ Path-I\left(\left|+z\right\rangle_p\right)\\ Path-II\left(\left|-z\right\rangle_p\right)\end{cases} \& \begin{cases}\alpha = 0\ \&\ \pi/2\left(\left|+x\right\rangle_s\ \&\left|+y\right\rangle_s\right)\\ up-spin\left(\left|+z\right\rangle_s\right)\\ down-spin\left(\left|-z\right\rangle_s\right)\end{cases}$$

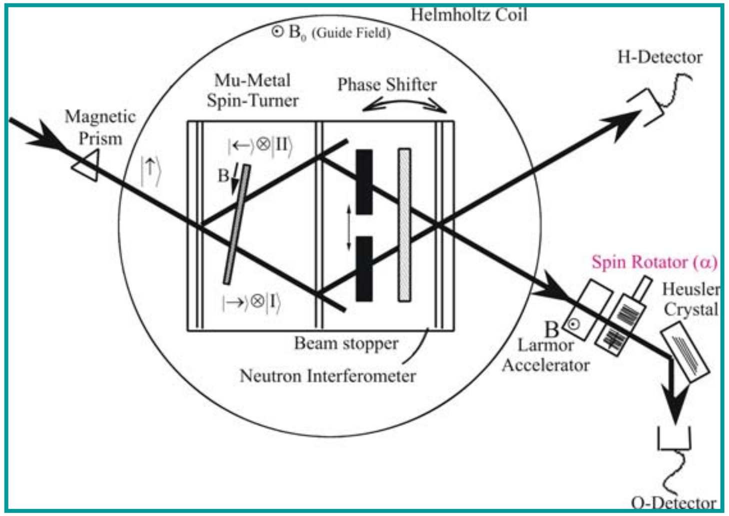
Typical results --- χ oscillations at $\sigma^{s}_{\pm z}$



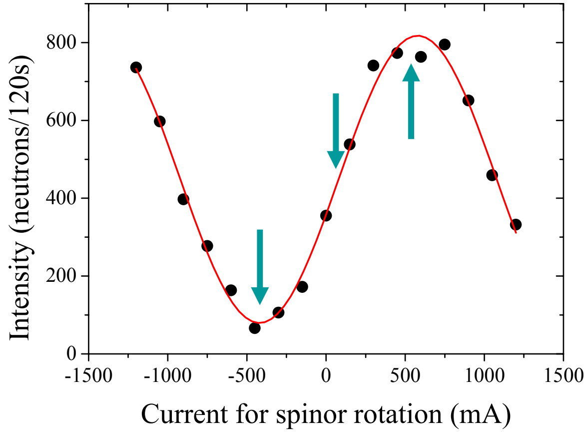


$$\left\{ \begin{array}{l} \langle \sigma^{p}_{+x} \cdot \sigma^{s}_{+z} \rangle \\ \langle \sigma^{p}_{+y} \cdot \sigma^{s}_{+z} \rangle \end{array} \right. \ \& \ \left\{ \begin{array}{l} \langle \sigma^{p}_{+x} \cdot \sigma^{s}_{-z} \rangle \\ \langle \sigma^{p}_{+y} \cdot \sigma^{s}_{-z} \rangle \end{array} \right. \text{ are determined.}$$

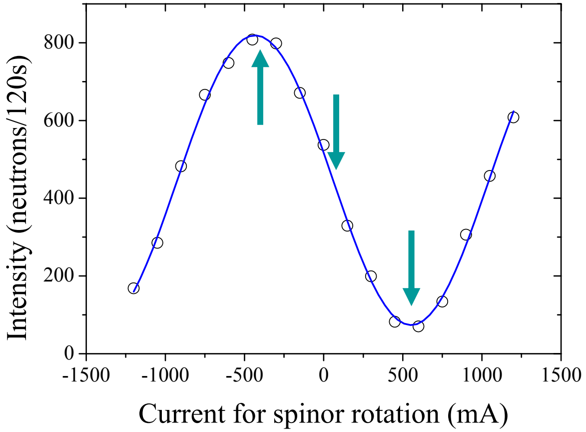
Typical results --- α oscillations at $\sigma^{p}_{\pm z}$



σ^{p}_{+z}



σ^{p}_{-z}



$\left\{ \begin{array}{l} \langle \sigma^{p}_{+z} \cdot \sigma^{s}_{+x} \rangle \\ \langle \sigma^{p}_{+z} \cdot \sigma^{s}_{\pm z} \rangle \end{array} \right. \;\&\; \left\{ \begin{array}{l} \langle \sigma^{p}_{-z} \cdot \sigma^{s}_{+x} \rangle \\ \langle \sigma^{p}_{-z} \cdot \sigma^{s}_{\pm z} \rangle \end{array} \right.$ are determined.

Analysis

$$(1)\left\langle \sigma^{s}_{+z}\cdot\sigma^{p}_{+z}\right\rangle = 0.543\times(1+0.069\cdot\sin 9^\circ),$$
$$(2)\left\langle \sigma^{s}_{+y}\cdot\sigma^{p}_{+z}\right\rangle = 0.543\times(1+0.069\cdot\cos 9^\circ),$$
$$(3)\left\langle \sigma^{s}_{+x}\cdot\sigma^{p}_{+z}\right\rangle = 0.501\times(1+0.820\cdot\sin 88^\circ),$$
$$(4)\left\langle \sigma^{s}_{-x}\cdot\sigma^{p}_{+z}\right\rangle = 0.501\times(1-0.820\cdot\sin 88^\circ),$$

$$(5)\left\langle \sigma^{s}_{+z}\cdot\sigma^{p}_{-z}\right\rangle = 0.457\times(1+0.122\cdot\sin 19^\circ),$$
$$(6)\left\langle \sigma^{s}_{+y}\cdot\sigma^{p}_{-z}\right\rangle = 0.457\times(1+0.122\cdot\cos 19^\circ),$$
$$(7)\left\langle \sigma^{s}_{+x}\cdot\sigma^{p}_{-z}\right\rangle = 0.499\times(1+0.833\cdot\sin 88^\circ),$$
$$(8)\left\langle \sigma^{s}_{-x}\cdot\sigma^{p}_{-z}\right\rangle = 0.499\times(1-0.833\cdot\sin 88^\circ),$$

$$(9)\left\langle \sigma^{s}_{+z}\cdot\sigma^{p}_{+x}\right\rangle = 0.473\times(1+0.706\cdot\cos 11^\circ),$$
$$(10)\left\langle \sigma^{s}_{+y}\cdot\sigma^{p}_{+x}\right\rangle = 0.473\times(1+0.706\cdot\sin 6^\circ),$$
$$(11)\left\langle \sigma^{s}_{+x}\cdot\sigma^{p}_{+x}\right\rangle = 0.506\times(1+0.025\cdot\sin 11^\circ),$$
$$(12)\left\langle \sigma^{s}_{-x}\cdot\sigma^{p}_{+x}\right\rangle = 0.494\times(1-0.025\cdot\sin 11^\circ),$$

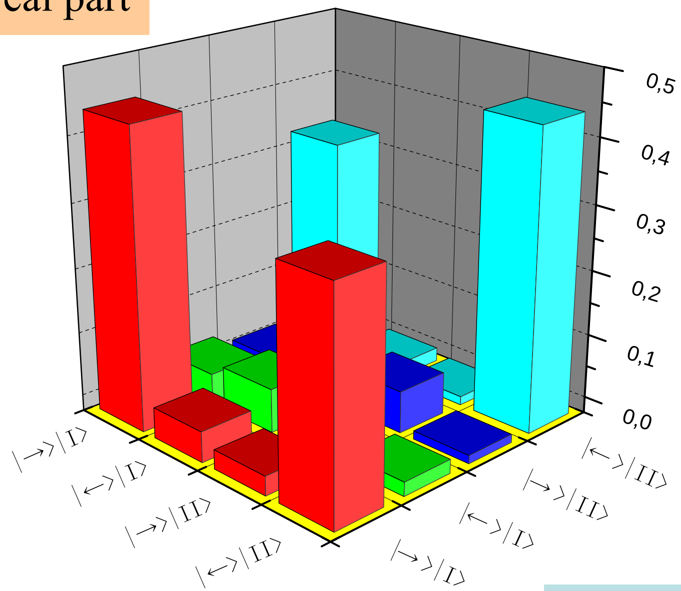
$$(13)\left\langle \sigma^{s}_{+z}\cdot\sigma^{p}_{+y}\right\rangle = 0.527\times(1+0.653\cdot\sin 11^\circ),$$
$$(14)\left\langle \sigma^{s}_{+y}\cdot\sigma^{p}_{+y}\right\rangle = 0.527\times(1+0.653\cdot\cos 6^\circ),$$
$$(15)\left\langle \sigma^{s}_{+x}\cdot\sigma^{p}_{+y}\right\rangle = 0.506\times(1+0.213\cdot\sin 11^\circ),$$
$$(16)\left\langle \sigma^{s}_{-x}\cdot\sigma^{p}_{+y}\right\rangle = 0.494\times(1-0.213\cdot\sin 11^\circ),$$

These 16-values → density matrix, ρ

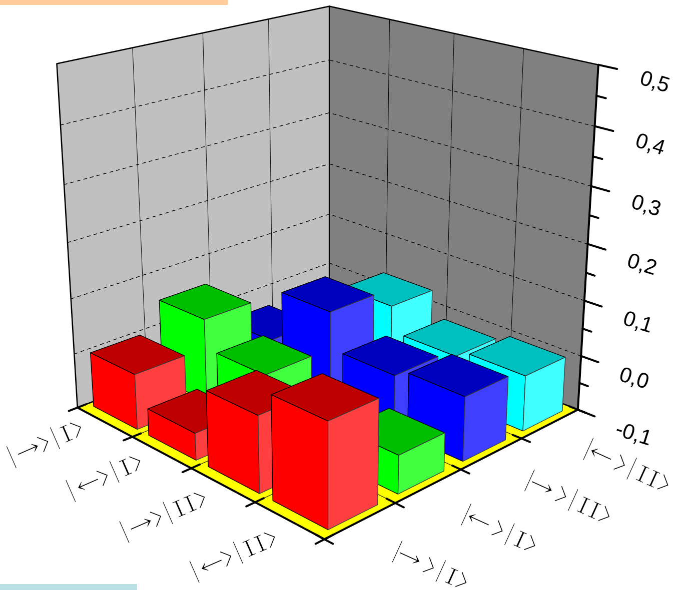
Quantum state tomography of neutron's Bell-state

$$|\Psi_1\rangle = |\rightarrow\rangle|\mathrm{I}\rangle + |\leftarrow\rangle|\mathrm{II}\rangle$$

real part



imaginary part



$$F = \langle\Psi|\rho|\Psi\rangle = 0.79$$

Schematic view of the experiment

◊ Incident $|\uparrow\rangle$, with spin-turner

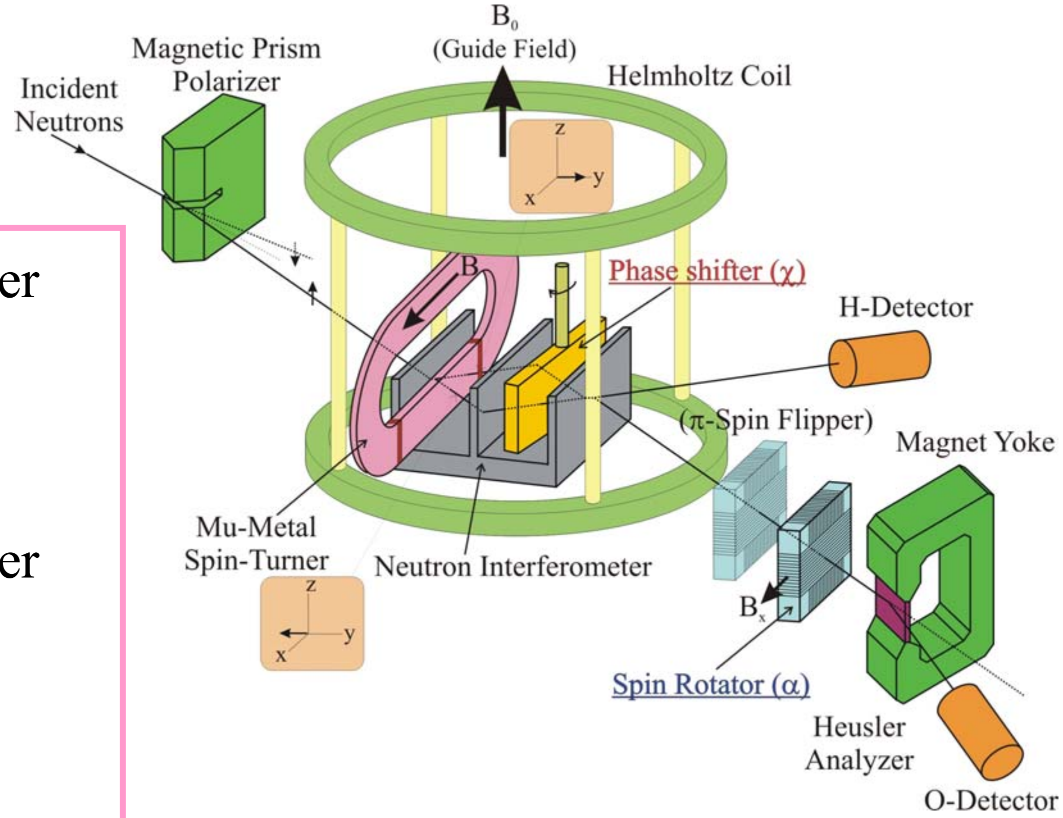
$$|\Psi_1\rangle = |\rightarrow\rangle|\mathrm{I}\rangle + |\leftarrow\rangle|\mathrm{II}\rangle$$

◊ Incident $|\downarrow\rangle$, with spin-turner

$$|\Psi_2\rangle = |\leftarrow\rangle|\mathrm{I}\rangle + |\rightarrow\rangle|\mathrm{II}\rangle$$

◊ Incident $|\uparrow\rangle$, without spin-turner

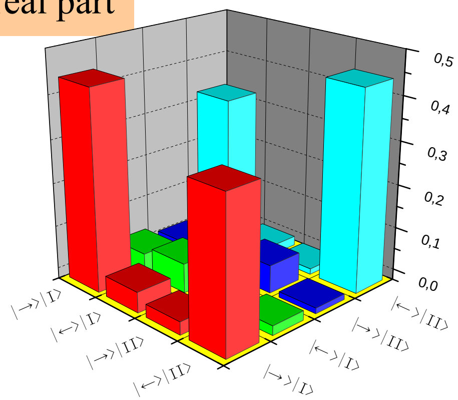
$$|\Psi_0\rangle = |\uparrow\rangle|\mathrm{I}\rangle + |\uparrow\rangle|\mathrm{II}\rangle$$

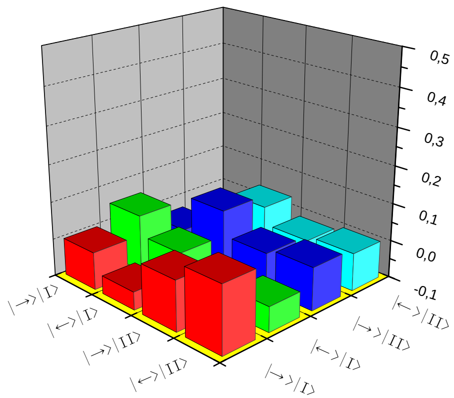


Quantum state tomography --- comparison

$|\Psi_1\rangle = |\rightarrow\rangle|I\rangle + |\leftarrow\rangle|II\rangle$

real part

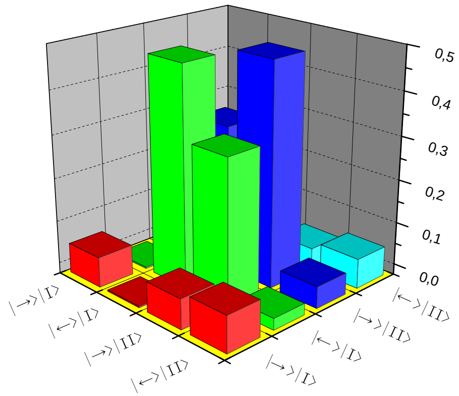


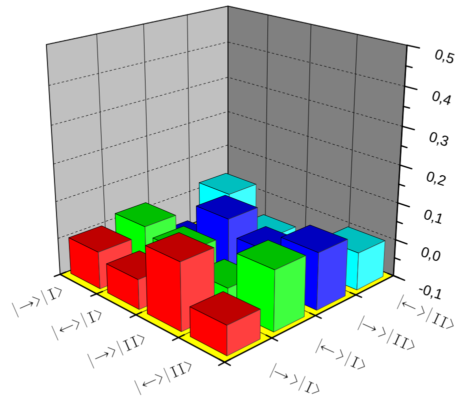


imaginary part

F=0.79

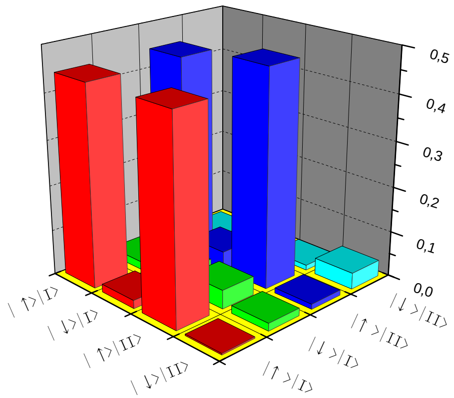
$|\Psi_2\rangle = |\leftarrow\rangle|I\rangle + |\rightarrow\rangle|II\rangle$

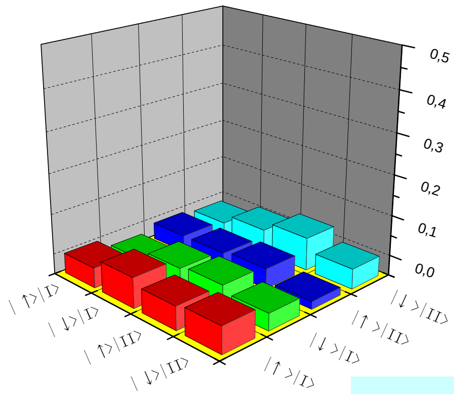




F=0.75

$|\Psi_0\rangle = |\uparrow\rangle|I\rangle + |\uparrow\rangle|II\rangle$





F=0.91

Neutron optical experiments exploring foundamental quantum-phenomena

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PRESTO, Japan Science and Technology Agency, JAPAN

1. Neutron interferometer/polarimeter

2. Recent neutron optical experiments

2-1 Quantum contextuality
- Violation of a Bell-like inequality
- Kochen-Specker-like paradox

2-2 Quantum state tomography

2-3 Geometric phase
- Non-cyclic spatial geometric phase
- Geometric phase for mixed state

3. Summary

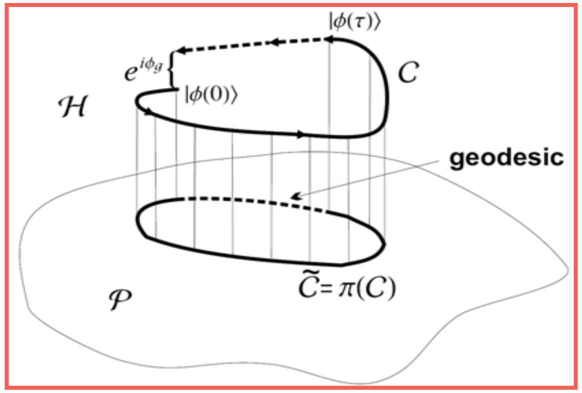
Geometric phase

Cyclic evolution: $|\Psi(T)\rangle = e^{i\phi} |\Psi(0)\rangle$

Pancharatnam ('56)
Berry ('84)
Simon ('83)
Aharonov & Anandan ('87)
Samuel & Bhandari ('88)

Rewriting with a periodic function, ($|\phi(T)\rangle = |\phi(0)\rangle$)

$|\Psi(t)\rangle = e^{if(t)} |\phi(t)\rangle$ with $f(T) - f(0) = \phi$



Multiplying and integrating the Schrödinger equation,

$$\int \langle \Psi(t)| \hat{H} |\Psi(t)\rangle \, dt = i\hbar \int \left\langle \Psi(t) \right| \frac{d}{dt} \left| \Psi(t) \right\rangle dt$$

Thus, one obtains

total phase

$$\phi = -\frac{1}{\hbar}\int_0^T \langle \Psi(t)| \hat{H} |\Psi(t)\rangle dt + i\int_0^T \left\langle \phi(t) \right| \frac{d}{dt} \left| \phi(t) \right\rangle dt$$

$$= \delta + \gamma$$

dynamical geometric

Geometric phase for 1/2-spin system

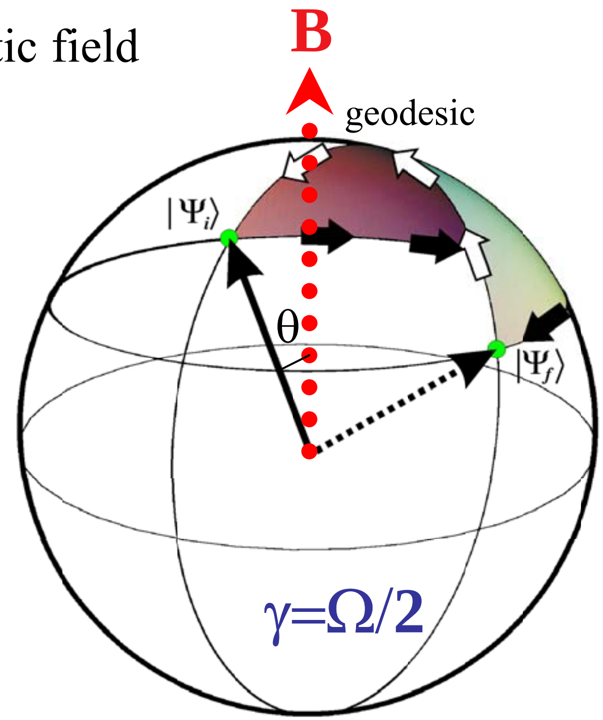
Example: 1/2-spin of neutrons in a magnetic field

$$|\Psi(T)\rangle = \begin{bmatrix} \exp\left(i\mu Bt/\hbar\right)\cdot\cos\left(\theta/2\right) \\ \exp\left(-i\mu Bt/\hbar\right)\cdot\sin\left(\theta/2\right) \end{bmatrix}$$

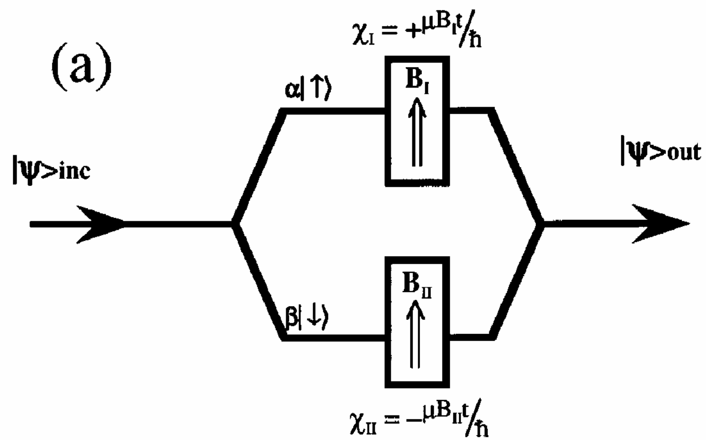
θ: polar angle from the direction of the magnetic field

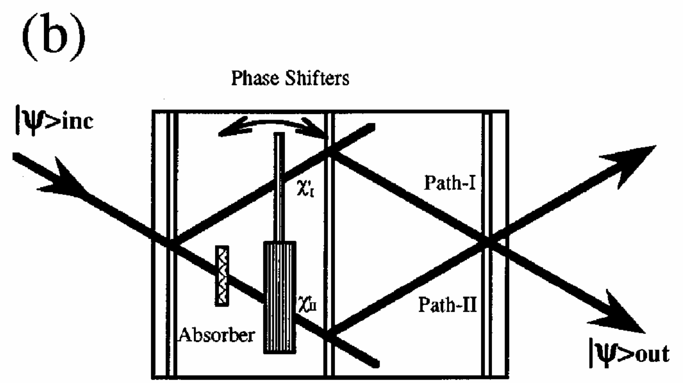
For a periodic evolution, $\left(T = n\pi\hbar/\mu B\right)$

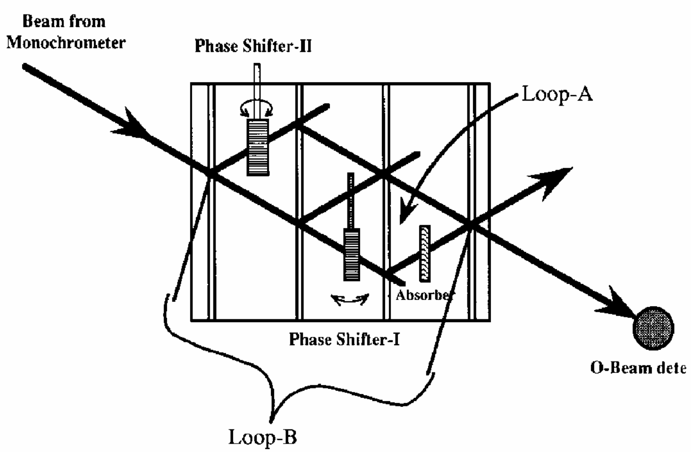
$$\begin{cases} \delta = n\pi\cdot\cos\theta & \text{dynamical} \\ \gamma = n\pi\cdot(1-\cos\theta) & \text{geometric} \end{cases}$$



Geometric phase in split-beam experiment







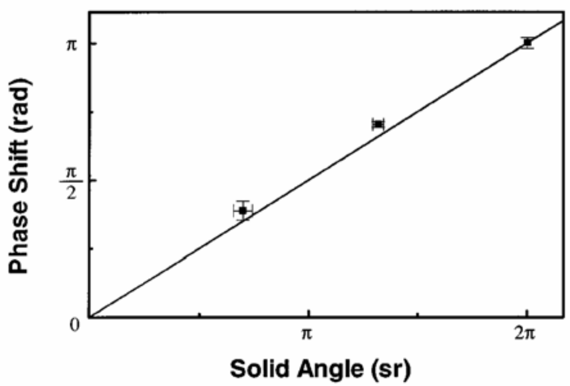
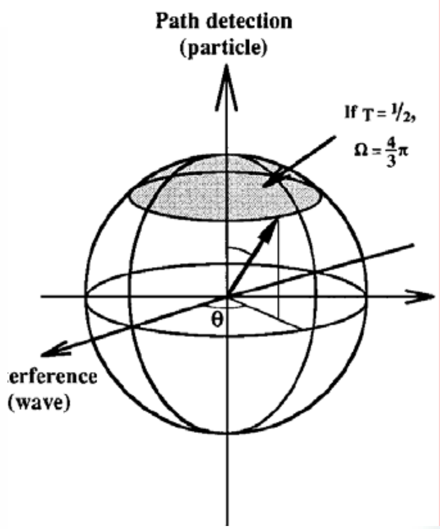


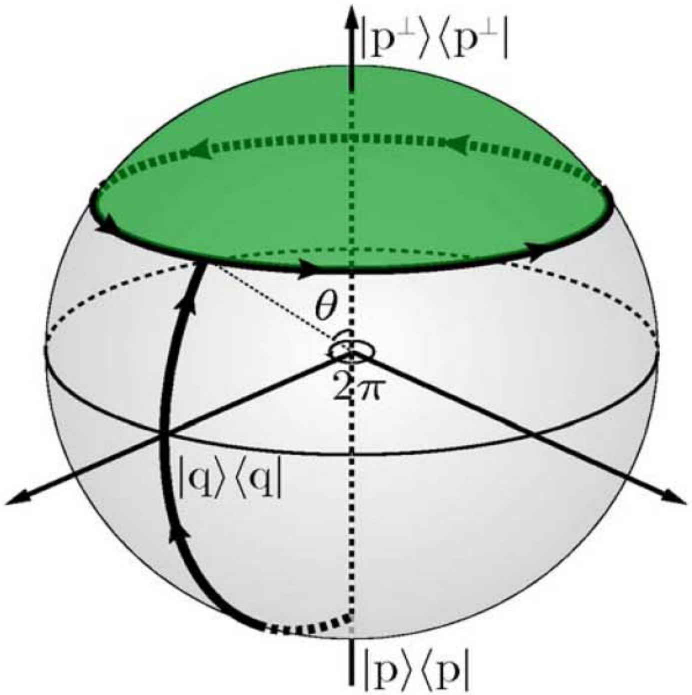
FIG. 7. Experimental results of geometric phase shift as a function of solid angle in the split-beam experiment. The solid line corresponds to the theoretical prediction given by Eq. (9) for $\sigma=\frac{1}{2}$.

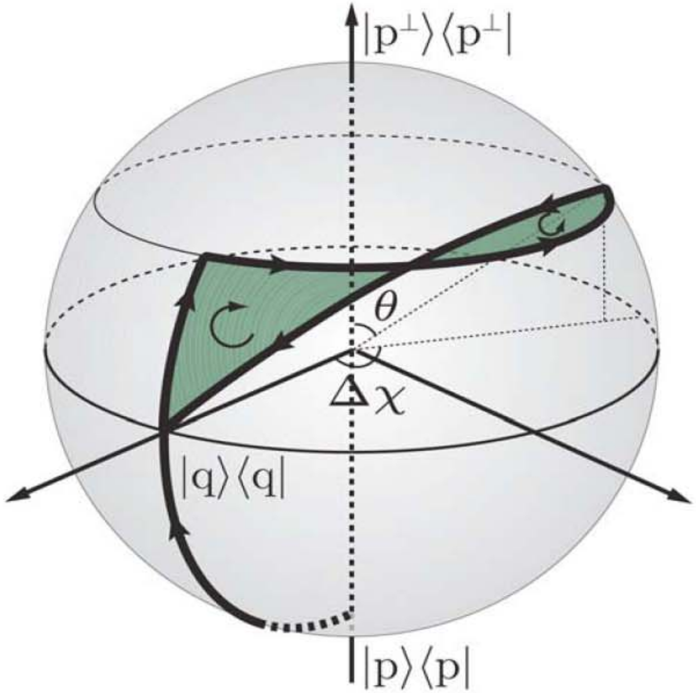


Y.Hasegawa et al., PRA**53** (1996) 2486.

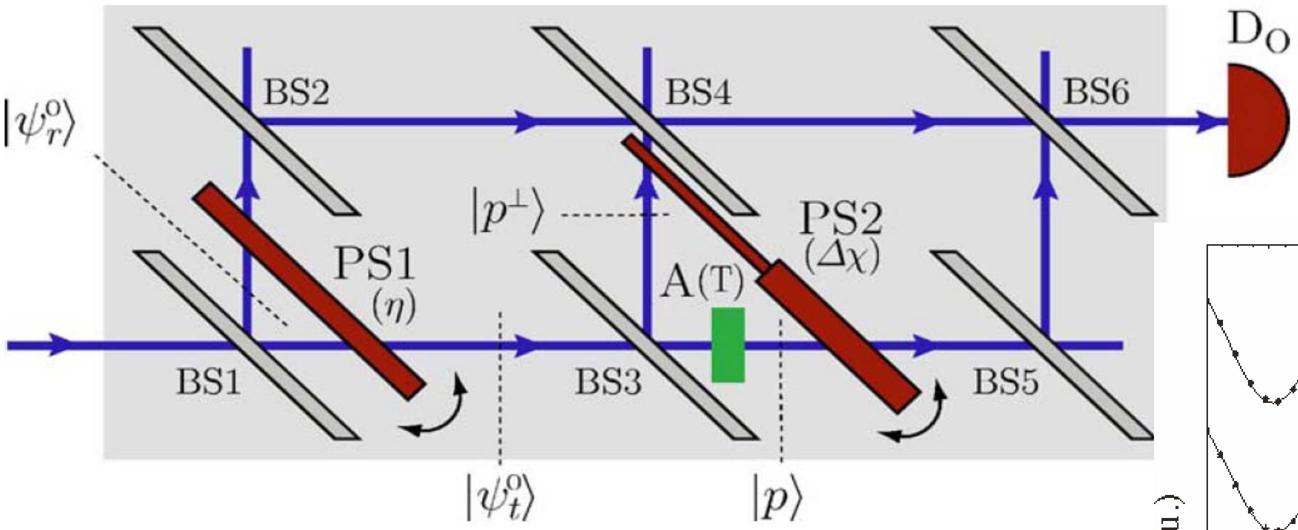
Non-cyclic evolution

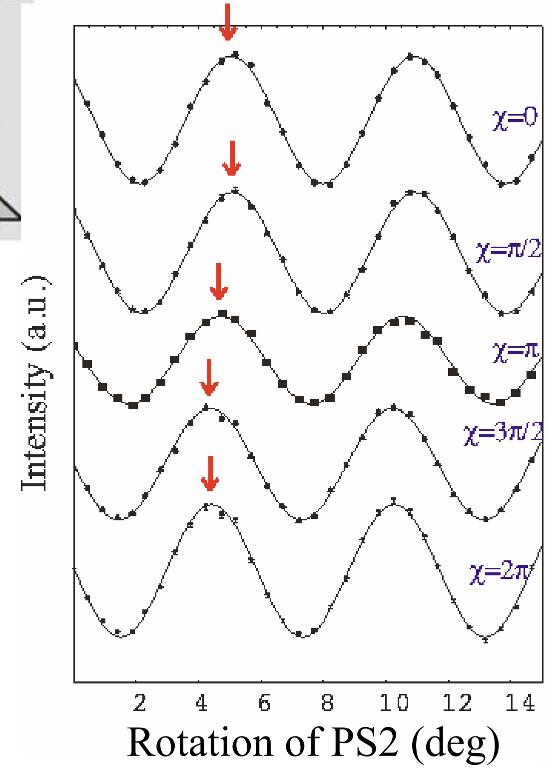
Cyclic evolution → ***Non***-cyclic evolution





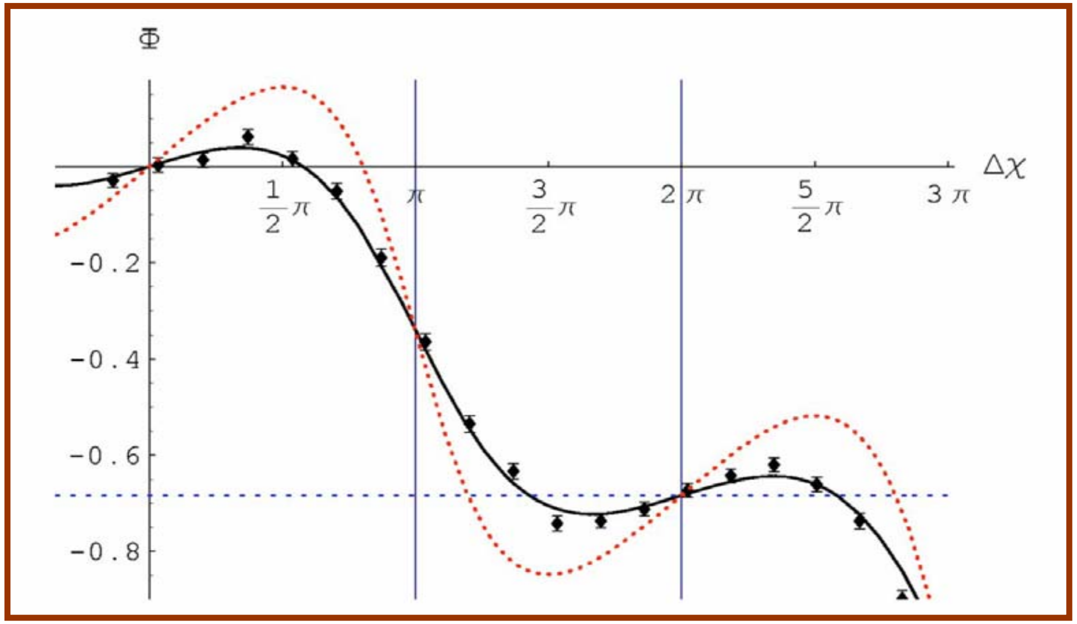
Experimental results (1)

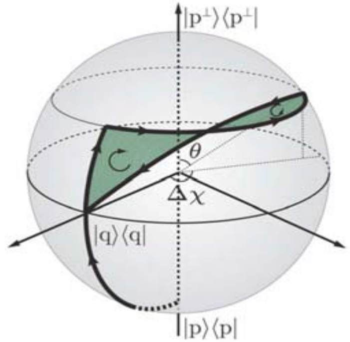


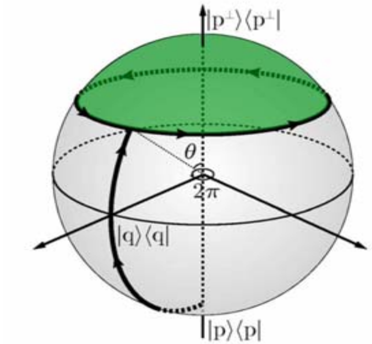


Experimental results (2)

Cyclic evolution → ***Non*-cyclic evolution**

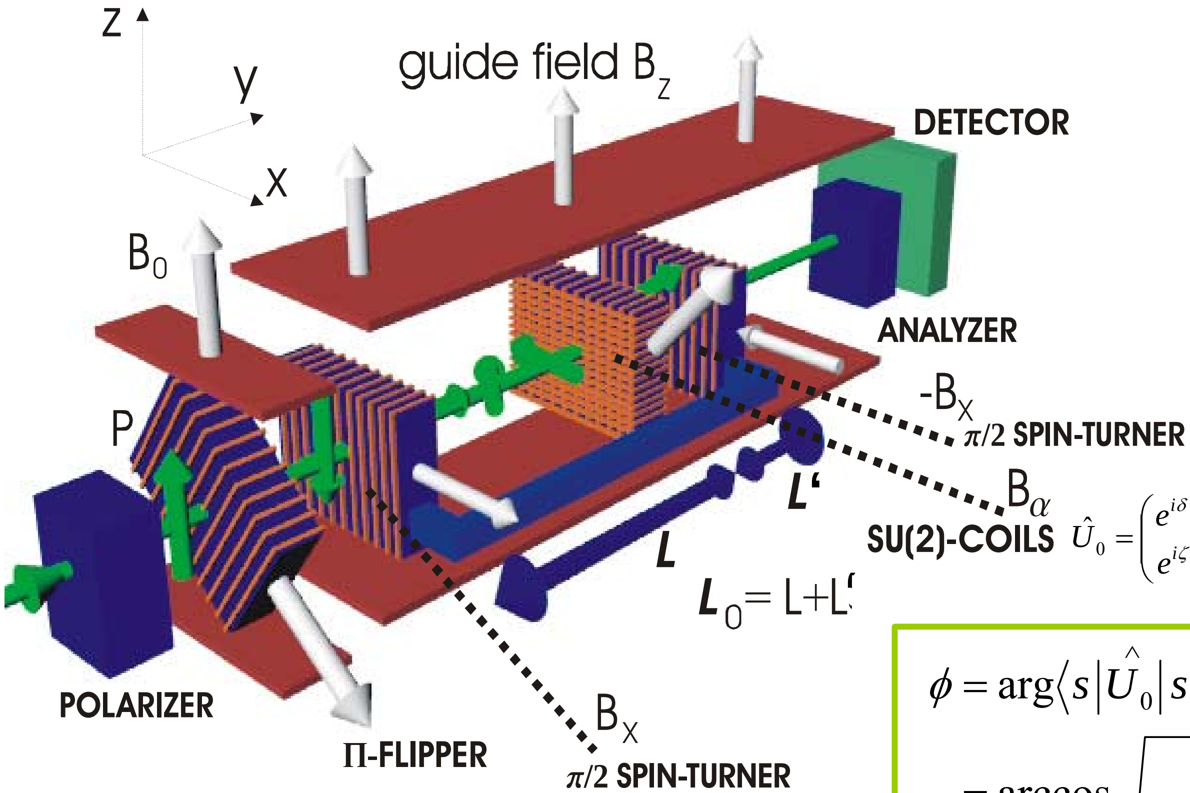






S. Filipp, Y. Hasegawa, R. Loidl, and H. Rauch, quant-ph/0412038; PRA72 (2005) 021602(R)

Geometric phase for mixed state



$$\hat{U}_0 = \begin{pmatrix} e^{i\delta}\cos\xi & -e^{-i\zeta}\sin\xi \\ e^{i\zeta}\sin\xi & e^{-i\delta}\cos\xi \end{pmatrix}$$

$$\phi = \arg\langle s|\hat{U}_0|s\rangle = \arccos\sqrt{\frac{I_{\min}}{I_0 - I_{\max} + I_{\min}}}$$

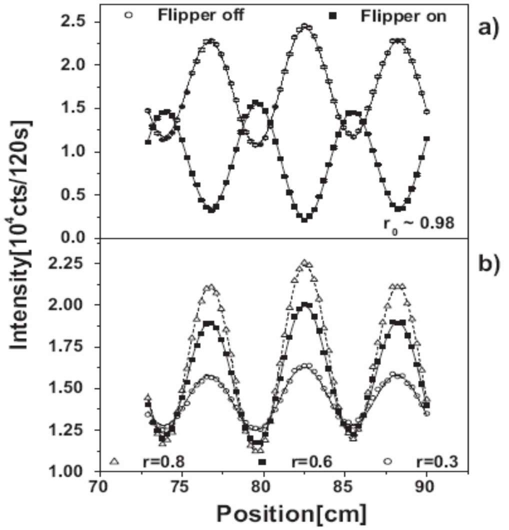
$$I = \left|\langle +z|\,\hat{U}_x(\pi/2)\,\hat{U}_z(\eta)\,\hat{U}_0\,\hat{U}_z(2\pi n - \eta)\,\hat{U}_x(-\pi/2)\,|+z\rangle\right|^2$$

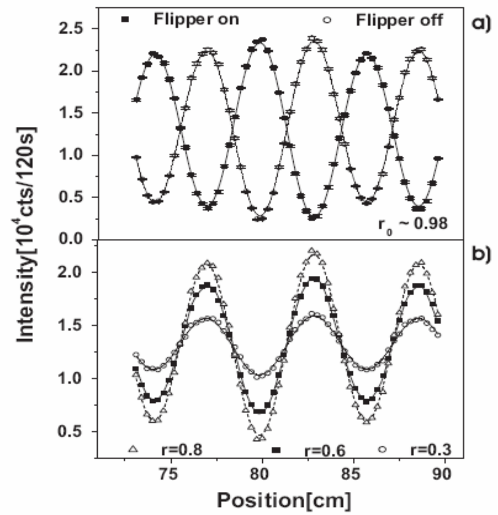
$$= \cos^2\xi\cos^2\delta + \sin^2\xi\sin^2(\zeta-\eta) \quad \Rightarrow \quad I_{\min} = \cos^2\xi\cos^2\delta,$$

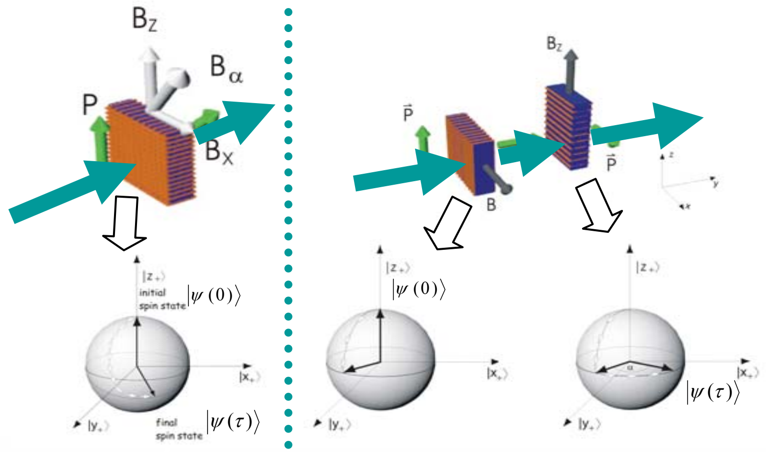
$$I_{\max} = \cos^2\xi\cos^2\delta + \sin^2\xi$$

J. Klepp, S. Sponar, Y. Hasegawa, E. Jericha and G. Badurek, quant-ph/0505209; PLA **342** (2005) 48.

Experimental results

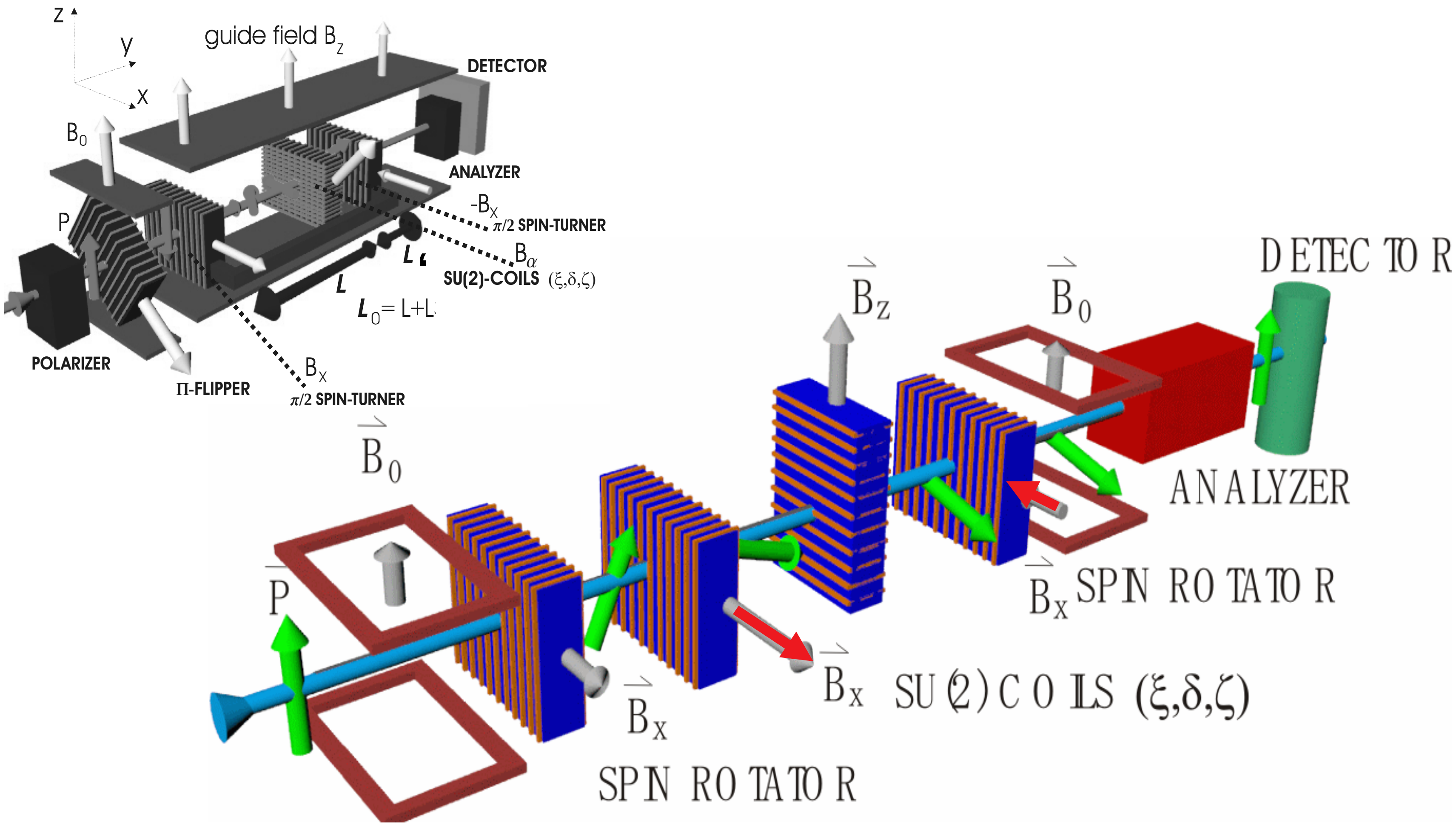






1)	r	0.976 ± 0.004	0.8	0.6	0.3
	Φ_{th}	0.37	0.30	0.23	0.11
	Φ_m	0.38 ± 0.08	0.35 ± 0.06	0.21 ± 0.06	0.05 ± 0.07
2)	r	0.981 ± 0.005	0.8	0.6	0.3
	Φ_{th}	0.17	0.14	0.10	0.05
	Φ_m	0.16 ± 0.06	0.12 ± 0.06	0.10 ± 0.07	0.02 ± 0.08

Work in progress --- new sophisticated setup



Work in progress --- production of mixed states

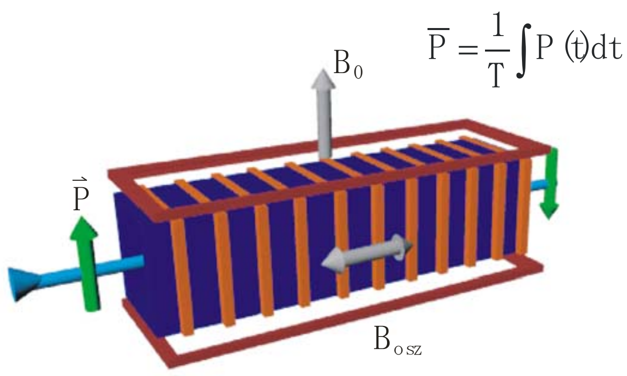
(1) Random arrival time: $\mathbf{B}(t_{rnd})$ ➔ $\mathbf{P}(t)$

(2) Random magnetic field: $\mathbf{B}_{rnd}(t)$ ➔ $\mathbf{P}(t)$

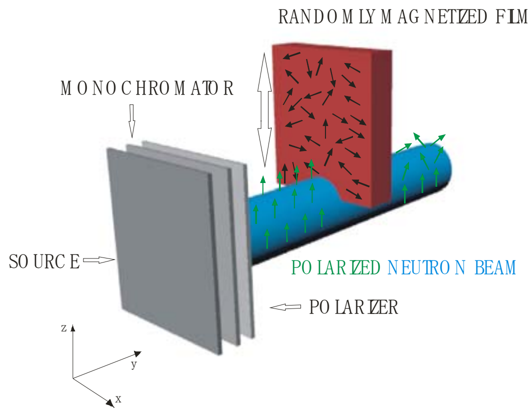
(3) Mixture of pure and complete mixed state

(Werner state analogy): $p\cdot|\Psi_{pure}\rangle\langle\Psi_{pure}|+(1-p)\cdot \mathit{diag}(1/2,1/2)$

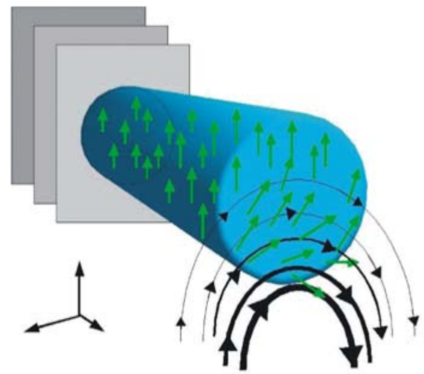
(4) Inhomogenous magnetic field: $\mathbf{B}(\mathbf{r})$ ➔ $\mathbf{P}(\mathbf{r})$



(1)



(3)



(4)

Summary

☆ Quantum contextuality: violation of a Bell-like inequality
Kochen-Specker-like paradox

☆ Quantum state tomography:
➔ These tomographical technique will be applied to,
- decoherence/depolarization process,
- characterization of robustness of geometric phase,
- non-unitary evolution of neutron's entangled state,
- etc.

especially from the polarized incident!

☆ Geometric phases:
➔ Error tolerant and robust phase

Fin !