COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Operational approach to the phase of a quantum field"

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(Received 18 February 1992)

Interpretational difficulties of a quantum measurement [J.W Noh, A. Fougères, and L. Mandel, Phys. Rev. A **45**, 424 (1992)] are pointed out. The nonunitarity of the respective exponential phase operator reflects an enhanced noise, which can be reduced by discarding ambiguous output data. Consequently, dispersion of such measurements does not represent a meaningful performance measure relevant to the input state. Moreover, the proposed model of quantum phase measurements provides a discrete spectrum of the quantum phase operator, whereas the phase-shift parameter is continuous.

PACS number(s): 42.50.Wm, 03.65.Bz

Recently Noh, Fougères, and Mandel (NFM) suggested and performed in Refs. [1] and [2] an interesting quantum measurement based on photon counting by four independent photon counters in the multiport homodyne scheme in Fig. 2 of Ref. [2]. They proposed to measure simultaneously the sine and cosine of the phase shift and use this directional data for a statistical description of the phase. The agreement between experiment and developed theory is convincing and therefore they proposed to identify this phase concept with the "correct" phase operator in quantum optics. The purpose of this Comment is to point out some flaws which hinder the interpretation of the suggested quantum measurement as the measurement of phase shift. My argumentation is based on the dispersion as the appropriate statistical measure of a phase variable. To judge the accuracy of available phase data according to mathematical statistics [3], the dispersion $D^2 = 1 - |\langle e^{i\phi}\rangle|^2$ has to be used. The respective mean value $\langle e^{i\phi}\rangle$ may be easily rewritten as the expectational value of the exponential phase operator. Even if this idea is not new in physics [4], dispersion is often used incorrectly [5] in the existing literature.

The NFM model is based on the measurement of the complex amplitude

$$\begin{aligned}\hat{Y}_{\rm NFM} &= \hat{n}_4 - \hat{n}_3 + i[\hat{n}_6 - \hat{n}_5] \\ &= \hat{a}_2\hat{a}_1^\dagger - \hat{a}_{20}\hat{a}_{10}^\dagger + i(\hat{a}_1\hat{a}_{20}^\dagger + \hat{a}_{10}\hat{a}_2^\dagger),\end{aligned} \tag{1}$$

and the quantum phase is then specified as the measurement of the phaselike parameter $\phi = \arg[Y]$. Before calculating the variance $V^2 = \langle(\Delta\cos\phi)^2\rangle + \langle(\Delta\sin\phi)^2\rangle$, the ambiguous (noisy) phase data are discarded. We will show that this treatment causes interpretational difficulties. Let us introduce the exponential phase operator

$$\hat{R} = \begin{cases} \hat{Y}\,(\hat{Y}^\dagger\hat{Y})^{-1/2} & \text{for } \mathcal{H}_0^\perp \\ 0 & \text{for all states in } \mathcal{H}_0, \end{cases} \tag{2}$$

$\mathcal{H}_0$ being the subspace spanned by the eigenstates with the zero complex amplitude $\{|y=0\rangle\}$ and $\mathcal{H}_0^\perp$ being its orthogonal complement. The dispersion for the phase measurement in a general quantum state $|\psi\rangle$ is consequently given as

$$\begin{aligned}D^2 &= 1 - |\langle\psi|\hat{R}|\psi\rangle|^2 \\ &= 1 - \left|\sum_{y\ (\neq 0)} e^{i\arg(y)}|\langle\psi|y\rangle|^2\right|^2\end{aligned} \tag{3}$$

and is equal to the variance $\langle(\Delta\cos\phi)^2\rangle + \langle(\Delta\sin\phi)^2\rangle$, since the real and imaginary parts of the complex amplitude commute $[\hat{Y}_{\rm NFM}, \hat{Y}_{\rm NFM}^\dagger] = 0$. Unfortunately, such a quantity is not registered in Refs. [1,2], but instead of this, the variance, after discarding and renormalization

$$V^2 = 1 - \frac{\left|\sum_{y\ (\neq 0)} e^{i\arg(y)}|\langle\psi|y\rangle|^2\right|^2}{(1-P_0)^2}, \tag{4}$$

is used, $P_0 = \sum_{y=0}|\langle\psi|y=0\rangle|^2$ being the probability of the occurrence of ambiguous phase data. The relation between the used variance V and the dispersion D is then given as

$$V^2 = 1 - \frac{1-D^2}{(1-P_0)^2}. \tag{5}$$

Variance V does not represent a meaningful performance measure of the phase measurement performed in the quantum state $|\psi\rangle$, because the procedure of discarding ambiguous data effectively changes the measured quantum state. Explicitly, a general quantum state may be represented as the superposition of two orthogonal contributions

$$|\psi\rangle = \alpha\, |y=0\rangle + \beta\, |\chi\rangle,$$

$$\langle\chi|y=0\rangle = 0, \quad |\alpha|^2 + |\beta|^2 = 1.$$

This specification tends to the dispersion

$$D^2 = 1 - |\langle\psi|\hat{R}|\psi\rangle|^2 = 1 - |\beta|^4 |\langle\chi|\hat{R}|\chi\rangle|^2, \qquad (6)$$

whereas the variance

$$V^2 = 1 - \frac{|\beta|^4}{(1-|\alpha|^2)^2} |\langle\chi|\hat{R}|\chi\rangle|^2 = 1 - |\langle\chi|\hat{R}|\chi\rangle|^2. \qquad (7)$$

We see that V is the dispersion of the phase measurement performed in the state $|\chi\rangle$, independently on the multiplier β. Discarding the ambiguous data tends effectively to the projection and normalization of the wave function into the subspace orthogonal to the wave functions with ambiguous phase. Consequently, V is the meaningful performance measure of the phase measurement in the state $|\chi\rangle$, but not in the state $|\psi\rangle$. The differences are apparent, when $|\psi\rangle \to |y=0\rangle$ as the limit $\beta \to 0$.

All this difficulty comes from the nonunitarity of the exponential phase operator $\hat{R}$ yielding some portion of the noisy (ambiguous) data. The potential danger of misinterpretation is in the fact that the discarded measurements are experimentally available, whereas the data without discarding are important for evaluation of the accuracy of phase-shift measurement. The noise associated with the nonunitarity of the exponential phase operator has to be included and only dispersions D without discarding deserve to be compared. Otherwise, phase measurements in *different quantum states* are considered. All these conclusions may be demonstrated with the results of NFM investigations.

(i) Quantum measurement evaluated in Figs. 3 and 4 of Ref. [2] cannot be treated as phase measurement in a coherent field with specified complex amplitude, especially this is significant in the case of an almost vacuum field when P_0 goes to 1.

(ii) The difference between V and D in the NFM phase concept seems to be negligible in the case of P_0 small as in Figs. 5 and 6 of Ref. [2]. Nevertheless problems occur for phase concepts with noncommuting operators associated with the sine and cosine of the phase, as in the case of the Susskind-Glogower (SG) phase operator. Using dispersion, the discrepancy between the Pegg-Barnett phase concept and the SG one in Fig. 6 of Ref. [2] disappear in accordance with Fig. 5 of Ref. [2] since both ideal models are physically equivalent [6]. Moreover, since the ideal phase measurement is the minimum dispersion estimator [7], the corresponding curve will be situated below the NFM one (see Fig. 1), in contradiction to Fig. 6 of Ref. [2]. Of course, this comparison has to be performed in the same quantum state, because both concepts—ideal or NFM—can be reduced in the same Hilbert space.

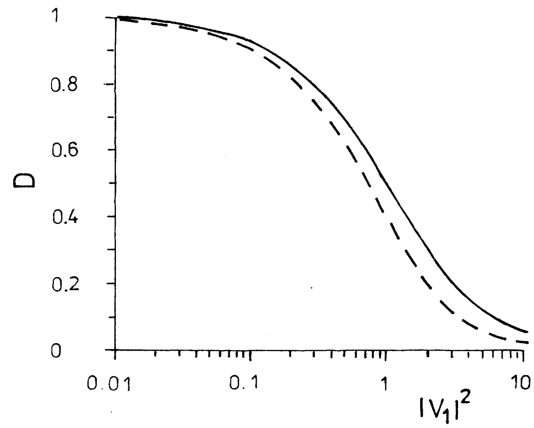

FIG. 1. Comparison of dispersions D for NFM (full line) and ideal (dashed line) phase concepts as functions of the mean detected photon number $|V_1|^2$. The quantum state is considered to be coherent and the NFM measurement is in accordance with Fig. 6 of Ref. [2].

(iii) The last objection is more fundamental. The purpose of any quantum phase operator is to provide the mesasurement of the phase-shift parameter, which can in principle achieve *continuously* each value in the interval $(-\pi, \pi]$. We expect therefore a *continuous* spectrum of the quantum phase operator. The NFM phase concept obviously provides the discrete spectrum and therefore does not fulfill this definition, at least in the setup discussed in Refs. [1,2]. This requirement of continuity seems to be reasonable, since otherwise dispersion is no more a performance measure relevant to the phase-shift variable. Especially, the zero dispersion may be achieved for an arbitrary nonzero energy, but the accuracy of available information about the phase shift is limited. I address this topic in a forthcoming paper.

In my opinion, the concepts of feasible quantum phase measurement need some portion of idealization in the sense that some auxiliary fields are considered as classical. The original idea of multiport homodyne detection is versatile enough to incorporate such treatment as the Shapiro-Wagner phase concept [8], which is feasible and provides a continuous spectrum.

I am grateful to R. Fernandez for valuable comments and a careful reading of the manuscript and to Professor A. Quattropani for the hospitality of the Institut de Physique Théorique EPFL.

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