Quantum optical experiments and fundamentals of quantum theory

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ABSTRACT

Quantum optics has offered new possibilities for experimental tests of basic principles of quantum mechanics. It enables us to experimentally investigate such phenomena as quantum interference and quantum non-locality. By means of cascade transitions or spontaneous parametric down-conversion, entangled photon pairs can be prepared. It makes possible to test the violation of Bell's inequalities and to study other issues connected with the question of completeness of quantum theory. The results of experiments of this kind pronounce in favour of quantum mechanics. The entangled pairs can further be used to demonstrate the violation of other classical inequalities, "non-local" interference in frequency domain, induced interference without induced emission, etc. They are also employed for quantum teleportation. Another family of interesting quantum optical experiments relates to single-photon interference. These experiments demonstrate wave-particle duality and correspondence between the interference visibility and the degree of knowledge of the photon's path in an interferometer. A spectacular example is the so-called interaction-free measurement. Quantum optical experiments open a new window on the quantum world and help us understand it. Further, quantum optics provides the ground for new interesting applications: quantum cryptography, quantum communications, and quantum computation.

Keywords: Quantum Optics, EPR Paradox, Bell's Inequalities, Quantum Non-locality, Down-conversion, Quantum Teleportation, Quantum Interference, Interaction-free Measurement

1. INTRODUCTION

Optics is one of the oldest disciplines of natural science. Nevertheless, it is still fresh, fruitful and progressive. Modern quantum optics offers large room for unique experimental tests of the fundamentals of quantum theory. Quantum optical experiments are relatively simple and graphic. These features could perhaps help us gain a better insight into quantum phenomena contradicting common sense.

In the following paragraphs we will discuss several optical experiments of such a kind. Of course, this brief review cannot be complete. Such exciting things like squeezed states of light (with their non-classical properties), cavity QED, quantum phase, reconstruction of quantum states, etc., will not be mentioned here (many of them can be found, e.g., in Refs.[1,2]). This text will be devoted to the question of local realism contra quantum theory, to Bell's inequalities and their experimental tests, quantum non-locality, quantum interference, parametric down-conversion, etc. "Fashionable" topics like quantum teleportation or interaction-free measurement will also be briefly discussed.

2. EPR "PARADOX"

Quantum theory can be viewed from different philosophical positions. One can accept a (more or less positivistic) attitude, that a theory represents just a set of relations between measurable quantities, and not to care about what, e.g., the wave function is, as it is cannot be measured directly. Besides, one can admit that the chance, i.e., probabilistic behavior, is inherent to microscopical phenomena and that there is no way to avoid it. Similar views were held by Niels Bohr (even if Bohr probably was not a positivist).

Such opinions are, however, very different from the "ideal of classical physics" defended by Albert Einstein. From Einstein's point of view, based on realism, a theory rather reflects behavior of real objects, whose existence is not brought into question (e.g., position and momentum have an objective meaning and coexist). The classical ideal is also strictly deterministic (exact knowledge of initial conditions enable us to exactly predict the results of

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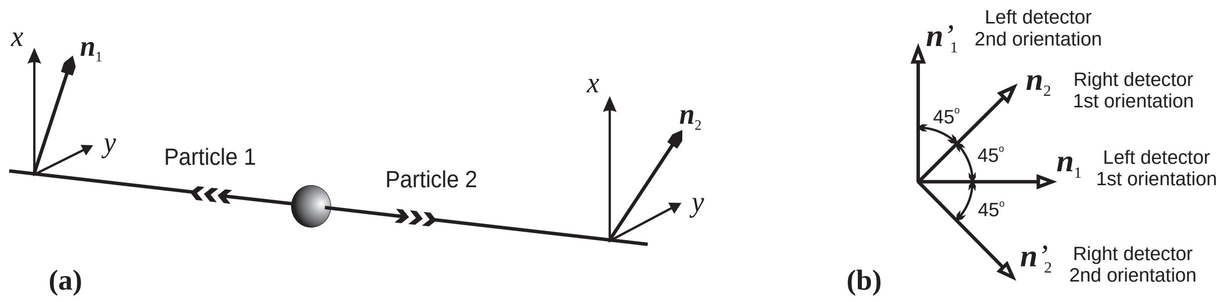

Figure 1. (a) Bohm's version of EPR gedanken experiment. (b) Special choice of orientations of Stern-Gerlach apparatuses.

any later measurements). From this position, quantum theory appears as an uncomplete, only temporary, theory, whose stochastic character reflects just our present ignorance of some hidden parameters. Extensive discussions between Bohr and Einstein about how to understand quantum mechanics brought in 1935 Einstein, together with B. Podolsky and N. Rosen, to the formulation of a gedanken experiment employing two particles prepared in a special state[3] to show the simultaneous existence of position and momentum, i.e., to demonstrate the overcoming of the uncertainty principle (this is why this experiment is called the EPR *paradox*). In 1952 David Bohm showed that the EPR gedanken experiment can also be reformulated for other non-commuting observables, namely for different spin projections.[4] It was the first step toward a realizable physical experiment.

Let us imagine a non-stable particle with spin 0, which has decayed to two particles with spins 1/2 propagating in opposite directions [see Fig. 1 (a)]. By means of a Stern-Gerlach apparatus, the projection of the spin of the first particle into the direction $\mathbf{n}_1$ is measured. Similarly, on the second particle the projection of the spin into the direction $\mathbf{n}_2$ is measured. The results of the measurements, when the spin projections are found in the positive course of the given directional vector ("spin up"), will be denoted by $+1$. Projections in the opposite course ("spin down") will have assigned the value -1. The quantum state of the two-particle system is described by the following state vector (the total angular momentum of the pair must equal zero):

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\mathbf{n},+\rangle_1 |\mathbf{n},-\rangle_2 - |\mathbf{n},-\rangle_1 |\mathbf{n},+\rangle_2 \right), \tag{1}$$

where the symbols $|\mathbf{n},+\rangle$ and $|\mathbf{n},-\rangle$ represent two orthogonal states of each individual particle with the positive ("up") and negative ("down") spin projections to an arbitrary direction given by a unit vector $\mathbf{n}$). This state vector can be written in no way as the product of two single-particle states – it is the so-called *entangled state*. Besides, it has the same form for any choice of $\mathbf{n}$. If the directions of orientations of both Stern-Gerlach apparatuses, $\mathbf{n}_1$ and $\mathbf{n}_2$, coincide, the measurement results are always perfectly (anti)correlated, i.e., $(+1,-1)$ or $(-1,+1)$.

EPR exponents assert that the quantum description is incomplete, that the results of measurements on both particles must already be given at the instant of the decay. The conception of EPR follows the next premises.

1. *Perfect correlation:* If the spins of both particles are measured along the same direction, the results will be opposite.
2. *Locality (separability):* If at the time of measurement the two subsystems (particles) do not interact, the measurement on one subsystem cannot affect, in any way, the other subsystem – at least it cannot do it immediately.
3. *Reality:* If, without disturbing the system, one can predict with certainty the value of a physical quantity, then there exists an *element of reality* corresponding to this quantity.
4. *Completeness:* Every element of reality must have a counterpart in a *complete* physical theory.

So, if one has measured, e.g., the positive value of spin projection of the first particle onto the x-axis (spin up), then he can predict with certainty – because of perfect correlation – that the measurement of the x-component of spin of the second particle must give the opposite result (spin down). However, at the last moment our experimentalist may decide to measure the y-component of spin of the first particle. Of course, the result "determines" the value of the spin projection onto y-axis of the second particle. As the particles cannot know what measurement one will decide to perform and as the measurement on the first particle cannot affect the state of the second one (the locality assumption), EPR infer that the elements of reality corresponding to spin projections both onto x and y directions must exist simultaneously. But it contradicts quantum mechanics, because the operators corresponding to x and y components of spin do not commute. EPR conclude that quantum mechanics is not a complete theory.

3. BELL'S INEQUALITIES

In 1964 John Bell shows that it is possible to arbitrate between the two above mentioned approaches (Bohr's and Einstein's) in a laboratory.[5]

Let us assume that the elements of reality corresponding to spin components really exist and that they are described by a hidden parameter (or a set of hidden parameters) λ which is not under our control. The parameter can randomly assume values from a set Λ with a probability measure ρ. Let us denote the response of the first Stern-Gerlach detector by $A(\mathbf{n}_1, \lambda)$ and the response of the second detector by $B(\mathbf{n}_2, \lambda)$. Both these functions may take on the values $+1$ or -1. The unit vectors $\mathbf{n}_1$ a $\mathbf{n}_2$ determine the orientations of detectors. Locality is involved in the fact that A does not depend on the orientation of the second Stern-Gerlach apparatus ($\mathbf{n}_2$) and B does not depend on $\mathbf{n}_1$. The correlation function (the mean value of the product) of functions A and B is then given by the expression

$$C(\mathbf{n}_1, \mathbf{n}_2) = \langle A(\mathbf{n}_1) B(\mathbf{n}_2) \rangle = \int_\Lambda A(\mathbf{n}_1, \lambda) B(\mathbf{n}_2, \lambda) d\rho. \tag{2}$$

Let us now consider four variables α, α', β and β' which may only assume values ± 1. Then the function

$$\gamma = \alpha\beta + \alpha\beta' + \alpha'\beta - \alpha'\beta' \tag{3}$$

can assume just the values $+2$ or -2 as one can simply ascertain realizing all 16 possible combinations. If we have a statistical ensemble of quaternaries $(\alpha, \alpha', \beta, \beta')$, then, obviously, the mean value of the function γ has to fulfill the following inequalities

$$-2 \leq \langle \gamma \rangle \leq 2. \tag{4}$$

Now, substituting $\alpha = A(\mathbf{n}_1, \lambda)$, $\alpha' = A(\mathbf{n}_1', \lambda)$, $\beta = B(\mathbf{n}_2, \lambda)$, and $\beta' = B(\mathbf{n}_2', \lambda)$, where $\mathbf{n}_{1(2)}$ and $\mathbf{n}_{1(2)}'$ represent two different orientations of the first and second detector, respectively, and using definition (2) we obtain the relation

$$|C(\mathbf{n}_1, \mathbf{n}_2) + C(\mathbf{n}_1', \mathbf{n}_2) + C(\mathbf{n}_1, \mathbf{n}_2') - C(\mathbf{n}_1', \mathbf{n}_2')| \leq 2. \tag{5}$$

Equation (5) represents one form of the famous *Bell's inequalities*.[6]

As we will see, quantum mechanics in general violates this inequality. Applying a proper transformation of the basis, state (1) can be rewritten in the form

$$\begin{aligned} |\psi\rangle = \frac{1}{\sqrt{2}} \big[&- (\sin\theta/2) |\mathbf{n}_1, +\rangle_1 |\mathbf{n}_2, +\rangle_2 + (\cos\theta/2) |\mathbf{n}_1, +\rangle_1 |\mathbf{n}_2, -\rangle_2 - \\ &- (\cos\theta/2) |\mathbf{n}_1, -\rangle_1 |\mathbf{n}_2, +\rangle_2 - (\sin\theta/2) |\mathbf{n}_1, -\rangle_1 |\mathbf{n}_2, -\rangle_2 \big], \end{aligned} \tag{6}$$

where θ is an angle between unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ (here it is assumed that both these vectors are lying in the plane perpendicular to the direction of propagation). From this expression it can be seen that the amplitude of probability to find the first particle with the positive value of its spin component in the direction given by $\mathbf{n}_1$ and, at the same time, the second particle with positive spin projection in the direction given by $\mathbf{n}_2$ is $-(\sin\theta/2)/\sqrt{2}$, etc. Thus the corresponding probabilities for different possible results are

$$\begin{aligned} P^\psi_{++}(\mathbf{n}_1, \mathbf{n}_2) &= P^\psi_{--}(\mathbf{n}_1, \mathbf{n}_2) = \frac{1}{2} \sin^2\theta/2, \\ P^\psi_{+-}(\mathbf{n}_1, \mathbf{n}_2) &= P^\psi_{-+}(\mathbf{n}_1, \mathbf{n}_2) = \frac{1}{2} \cos^2\theta/2. \end{aligned} \tag{7}$$

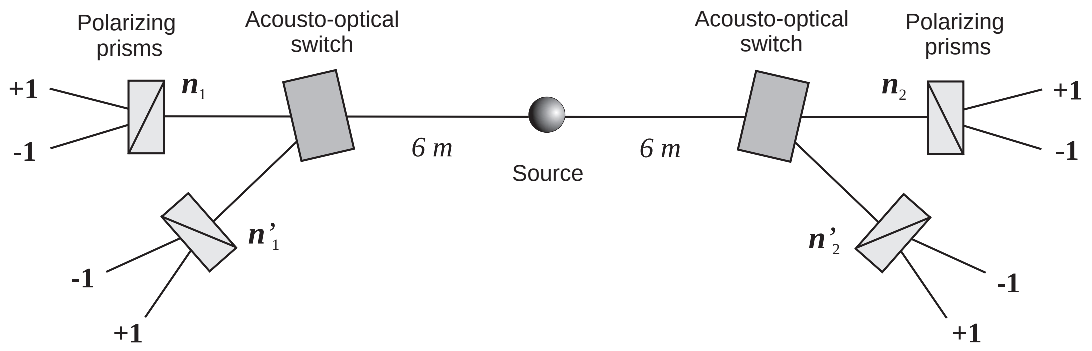

Figure 2. Outlines of the Orsay experiment.

The first index denotes the value of the projection of spin of the first particle in the direction $\mathbf{n}_1$, analogously the second index refers to the second particle and the second directional vector. From Eqs. (7), the quantum mechanical expression for the correlation function directly follows:

$$C(\mathbf{n}_1, \mathbf{n}_2) = \sin^2\theta/2 - \cos^2\theta/2 = -\cos\theta = -\mathbf{n}_1 \cdot \mathbf{n}_2. \tag{8}$$

We can now choose the directional vectors appearing in Eq. (5) in such a way that $\mathbf{n}_2$ with $\mathbf{n}_1$, $\mathbf{n}_1$ with $\mathbf{n}_2'$, and $\mathbf{n}_1'$ with $\mathbf{n}_2$ make an angle 45°, whilst $\mathbf{n}_1'$ with $\mathbf{n}_2'$ 135° [Fig. 1 (b)]. Then we can make four sets of measurements with different combinations of detector orientations. Quantum mechanics [see Eq. (8)] gives the following prediction for correlation functions of the measurements' results

$$\begin{aligned} &|C(\mathbf{n}_1, \mathbf{n}_2) + C(\mathbf{n}_1', \mathbf{n}_2) + C(\mathbf{n}_1, \mathbf{n}_2') - C(\mathbf{n}_1', \mathbf{n}_2')| = \\ &= |-\cos(45°) - \cos(45°) - \cos(45°) + \cos(135°)| = 2\sqrt{2} > 2. \end{aligned} \tag{9}$$

Ergo such orientations of Stern-Gerlach apparatuses exist, for which the prediction of quantum mechanics *violates* Bell's inequalities (inequalities that must be fulfilled by *any* local realistic theory)! This means that between quantum mechanics and the whole class of classical theories with hidden variables it can be arbitrated experimentally.

Note that even if quantum mechanics assumes a measurement on a single particle pertaining to an entangled pair to cause a state reduction of the whole system (including the other particle), its predictions concerning measurable quantities do not break causality. Nor quantum mechanics allows superluminal communications.

Till now we have discussed $\frac{1}{2}$-spin particles (mostly because of tradition). However, everything written in this section can be straightforwardly modified for the case of measurement of, e.g., linear polarizations of photons. Here we enter the land of optical experiments.

4. EXPERIMENTAL TESTS OF BELL'S INEQUALITIES

The first experimental tests of Bell's inequalities were performed by Freedman and Clauser in Berkeley in 1972.[7] They employed photons with correlated polarizations produced by cascade transitions in calcium atoms. Gradually more experiments appeared (some of them used γ-photons radiated during the annihilation of an electron and a positron, nevertheless cascade atomic decays prevailed): The experiments of Frye and Thompson (1975), Aspect and his group in Orsay (1976-1983), Grangier and Roger (1981, 82), or Ou and Mandel (1988) – to mention at least a few out of many experiment teams. Already the earliest experiments mostly approved the predictions of quantum mechanics. Their results, however, were not sufficiently confirmative . The violation of Bell's inequalities was demonstrated by only a few standard deviations, detector efficiencies were small, and the setting of orientations of polarizers was fixed for large sets of measurements. But the quality of results quickly increased.

Let us now focus our attention on the experiments performed by Aspect's group,[8] as they belong to the best elaborated ones. As the source of photon twins, a beam of calcium atoms excited by two lasers was used. Photons with entangled polarizations were produced by cascade decays (J denotes the angular momentum of the atom):

$$(J = 0) \overset{551.3\ \mathrm{nm}}{\longrightarrow} (J = 1) \overset{422.7\ \mathrm{nm}}{\longrightarrow} (J = 0).$$

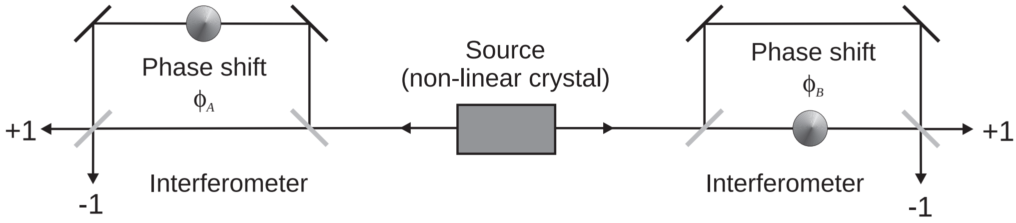

Figure 3. Setup for testing Bell's inequalities by means of two-photon interference.

In order to avoid the potential possibility that the setting of the polarization analyzer (a polarizing Wollaston prism) in one arm affects the situation in the other arm or even the process of pair preparation in a classical way – i.e., by some interaction occurring between the two detection apparatuses (and the source), the experiment was arranged in the following way: The angles of the polarization analyzers were set after the emission of the pair so that no classical signal (travelling at most as fast as light) could deliver information about the orientation of the first polarizing prism to the second one before the time of measurement. In practice it was done by two acousto-optical switches located in both arms. Each switch directs the photon to one of two differently oriented polarizing prisms (see Fig. 2). Acousto-optical switches were driven by two independent oscillators (one at each arm) with an average period of about 10 ns. Their distance from the photon source was 6 m – this corresponds to propagation time about 20 ns. In this sophisticated experiment the violation of Bell's inequalities was proven by more than 5 standard deviations. Nevertheless, in technically less complicated experiments without fast switches, the violation was demonstrated by several tens of standard deviations. New, very precisely elaborated tests of Bell's inequalities are prepared in Innsbruck[9] (as the source of entangled photons serves the so-called down-conversion in this case).

5. DOWN-CONVERSION

As already mentioned, entangled photons can also be produced in non-linear optical media (e.g., in crystals $KNbO_3$, $LiIO_3$, $LiNbO_3$, β-BaB_2O_4, etc.) by the process of spontaneous parametric frequency down-conversion. In this process, one photon from a pump laser is converted, with a certain probability, into two subfrequency photons. The total energy and momentum are conserved thereat. Since no couple of possible frequencies and wave vectors of two generated photons is preferred the resulting quantum state is given as a superposition of all allowed cases. Sources of this kind are very perspective since their practical realization is relatively simple. Using this technique, it is possible to prepare pairs with entangled polarizations discussed above, but in the first place these photon twins exhibit strong correlations of energies (frequencies) and directions of propagation. These types of quantum correlations can be utilized in many interesting "non-local" interference experiments. Bell's inequalities can be translated to the language of interferometric measurements and can be tested using energetically entangled states*.

6. INTERFEROMETRIC TESTS OF BELL'S INEQUALITIES

The scheme of an experiment for testing Bell's inequalities by means of "two-photon" interference (the 4th order interference) is sketched in Fig. 3. This configuration was proposed by Franson in 1989.[10] The source of entangled photons is a non-linear crystal pumped by a monochromatic short-wavelength laser. Generated subfrequency photons may be launched into optical fibres, e.g. Each of them is led to one of two unbalanced Mach-Zehnder interferometers. The path differences of the arms of both interferometers are the same and are chosen much greater than the coherence length of the individual photons. The setting of the additional phase difference (the fine path difference) is done by phase modulators placed in both interferometers. They induce phase shifts ϕ_A and ϕ_B. Since the path difference of arms in each interferometer is greater then the coherence length of light, one-photon interference cannot be observed. However, it can be shown that if the phase shifts ϕ_A and ϕ_B are the same†, then in the coincidence detection the photons will appear in the same output ports of the interferometers; i.e., observers at both parts obtain either both

*The Innsbruck experiment mentioned at the end of the preceding section uses polarization entanglement and standard form of the Bell's inequalities.

†Besides, $\Delta t\, \omega_0$ must be an integer multiple of 180°, where Δt denotes the difference of transit times in short and long arms of the interferometers and ω_0 is the frequency of the pump laser.

+1 or both −1. If ϕ_A and ϕ_B differ by 180°, the opposite situation occurs – the observers will obtain opposite results. For other phase-shift differences, both potentialities can randomly occur. We have met a similar situation with measurements of spin projections. In the configuration described here, the phase shifts ϕ_A and ϕ_B play the same role as the orientations of Stern-Gerlach apparatuses or polarizers in the methods discussed earlier. There is a strong time correlation between energetically entangled photons, but the moment of detection itself is completely uncertain. So, if we observe an exact time coincidence we cannot discern whether both photons passed through the short arms of interferometers or both photons passed through the long arms. This indistinguishability of paths can be interpreted as the reason for interference.

Fibre optics technology enables tests of Bell's inequalities at many kilometers.[11] The configuration described can also directly be used for quantum cryptography.[12]

7. NON-CLASSICAL PROPERTIES OF ENTANGLED PHOTON PAIRS

7.1. Violation of other classical inequalities

Photon pairs produced by parametric down-conversion have many other interesting non-classical features enabling large scale of different optical experiments. For instance, the violations of other inequalities valid for classical electromagnetic field can be demonstrated. E.g., the two beams generated by down-conversion violate the inequality

$$\langle I_1 I_2 \rangle \leq \frac{1}{2}(\langle I_1^2 \rangle + \langle I_2^2 \rangle), \tag{10}$$

where $\langle \ldots \rangle$ means averaging and I_1, I_2 denotes intensities of radiation. Correlation function $\langle I_1 I_2 \rangle$ is proportional to coincidence rate (relative count of simultaneous detections on both detectors). Reformulating this inequality for corresponding measurable quantities (real coincidence counters have a finite time resolution), its violation can be experimentally demonstrated. Zou, Wang, and Mandel did it by 600 standard deviation.[13]

7.2. "Non-local" interference in frequency domain

Correlations between frequencies of the entangled photons produced by down-conversion can be demonstrated in the experimental configuration proposed in Ref.[14]. If the path difference of the arms in a Mach-Zehnder interferometer exceeds the coherence length of radiation, the (second order) interference pattern cannot be seen. Nevertheless, the interference phenomena are latent in modulation of the spectrum of radiation. Placing appropriate frequency filters in front of detectors this modulation can be observed. Likewise, a filter, selecting only a narrow band of wavelengths, placed in front of the input port of the interferometer prolongs the coherence length of incident light and then the interference pattern at the output appears again. In the case of an entangled pair, however, the filter can be placed in the path of the "second" photon (which is not passing through the interferometer). During coincidence measurements one can yet observe interference effects (at the output of the interferometer mounted in the path of the "first" photon) dependent on the wavelength transmitted by the filter.

7.3. Time separation of photon twins

Another interesting feature of pairs of photons produced by down-conversion is their strong time correlation. Their maximal time separation is reciprocal to the frequency bandwidth. In practice, it is in a subpicosecond range, which is far under the resolution of contemporary detectors and electronics. However, the small variances from the perfect time correlation can be measured by means of interference techniques. The results of Hong, Ou, and Mandel[15] show that the time separations of entangled photons produced in a non-linear crystal do not exceed 100 fs. It is worth stressing again that the moments of detections themselves are completely uncertain. Focussing on the subsystem containing only one photon from the pair, we find that it is described by a density matrix which has the diagonal form in energetic representation – this correspond to a statistical mixture of monochromatic photons.

7.4. Induced interference without induced emission

A nice demonstration of connection between path indistinguishability and interference is given in an interference experiment with two nonlinear crystals.[17] Its scheme is in Fig. 4. Two non-linear crystal pumped by UV-laser are arranged in such a way that one of the output paths from the first crystal goes through the second crystal and is precisely aligned with one of the paths leaving the second crystal. The remaining two outputs of crystals are combined at a beam splitter. Changing slightly the position of the beam splitter, the phase shift between the two

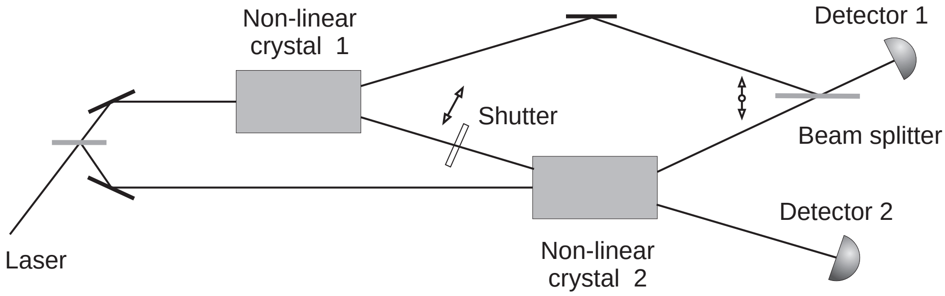

Figure 4. Interference experiment with two sources of entangled photons

beams can be varied. As the efficiency of down-conversion process is small, in the system there is very likely only one photon pair at a moment. So the cases of induced emission in the second crystal can be neglected. Nevertheless, the principal indistinguishability from which crystal the photon came induce (second order) interference behind the beam splitter. The detection rate (at detector 1) periodically changes when changing the position of the beam splitter. If, however, the joint of the crystals is broken (by a shutter), interference disappears. In a such case it is possible to distinguish, by means of detector 2, in which crystal the pair arose.

8. SYSTEMS OF THREE AND MORE ENTANGLED PARTICLES

Quantum entanglement is not limited to only two particles in any way. The system of three and more entangled particles have interesting features regarding the test of local realistic theories. These features have been studied by Greenberger, Horne, and Zeilinger.[18] Let us have a three-particle entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_1|+\rangle_2|+\rangle_3 + |-\rangle_1|-\rangle_2|-\rangle_3\right), \tag{11}$$

where $|\pm\rangle_j$ $(j = 1, 2, 3)$ denote two orthogonal states of individual particles corresponding, e.g., to two opposite values of z-component of spin (for $\frac{1}{2}$-spin particles). Let us assume that the detectors' outputs assume values ± 1 accordingly to whether the positive or negative projection of spin in the direction given by the azimuthal angle ϕ_j (in x-y plane) is measured. Quantum mechanics predicts that the mean value of the products of results of the three detectors is equal to $\cos(\phi_1 + \phi_2 + \phi_3)$. Thus for $\phi_1 + \phi_2 + \phi_3 = 0$ perfect correlation appear $(+1)$ and for $\phi_1 + \phi_2 + \phi_3 = \pi$ perfect anticorrelation (-1). If one attempts to introduce local realistic model with hidden variables, as was done for two particles, requiring also fulfilment of the perfect correlation (and anticorrelation) described above, he will run into problems. Each such model appears to be intrinsically *inconsistent*.

9. ENTANGLEMENT SWAPPING

Next I will briefly describe how to prepare multi-particle entangled states (necessary for testing Greenberger, Horne, Zeilinger predictions) by correlating photons from independent sources.[19] Let us have two independent sources of entangled pairs of particles, e.g., photons, producing two pairs (each of them one pair) at the same time. Quantum states of these two pairs will be described by the expressions $2^{-1/2}(|a\rangle_1|b\rangle_2 + |a'\rangle_1|b'\rangle_2)$ [the first one] and $2^{-1/2}(|c\rangle_3|d\rangle_4 + |c'\rangle_3|d'\rangle_4)$ [the second one]; i.e., photon 1 is correlated with photon 2 and photon 3 with photon 4[‡]. The total state of all four photons is then

$$|\psi_{\rm in}\rangle = \frac{1}{2}(|a\rangle_1|b\rangle_2 + |a'\rangle_1|b'\rangle_2)(|c\rangle_3|d\rangle_4 + |c'\rangle_3|d'\rangle_4). \tag{12}$$

If one combines – by means of a beam splitter – outputs c and b, and by another beam splitter outputs c' and b', and places a detector behind each of these beam splitters then, in the case of coincident detection of one photon at

[‡]Particular one-particle states may correspond, e.g., to different directions of propagation behind a non-linear crystal; withal, each couple of directions $[(a, b), (a', b')$ etc.$]$ must agree with momentum conservation.

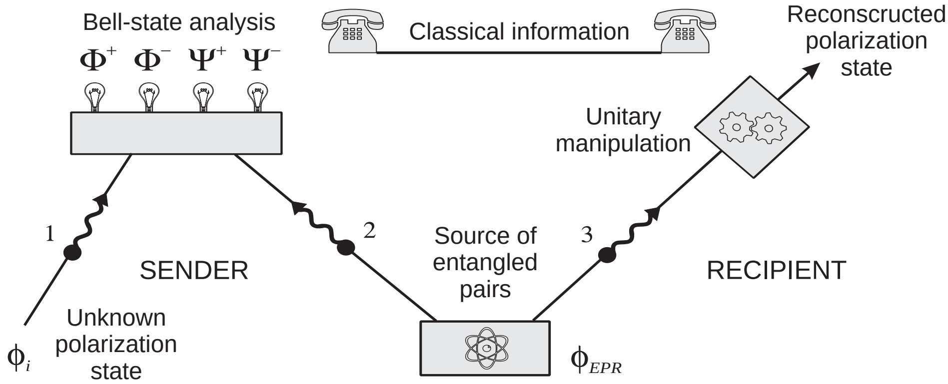

Figure 5. Quantum teleportation.

the first and one photon at the second detector, the quantum state of the remaining two photons collapses to

$$\frac{1}{\sqrt{2}}\left(|a\rangle_1|d'\rangle_4 + |a'\rangle_1|d\rangle_4\right), \tag{13}$$

i.e., photons 1 and 4 become entangled. In order this method could really work both photons must be registered by detectors in a time interval shorter then their correlation time. The correlation time is, as we know, of the order of hundreds of femtoseconds. Such precision is not accessible for contemporary detection technology. A solution is to synchronously pump both non-linear crystals by ultrashort optical pulses. Then the instants of the births of both photon pairs cannot differ by more than the pulse width.

In a completely analogous way – using more pulse-pumped non-linear crystals – it is possible to prepare multi-particle entangled states.

10. QUANTUM TELEPORTATION

Non-local quantum correlations represent a miraculous phenomenon, which (among others) enables us to realize no less interesting methods of quantum communications, namely quantum teleportation, quantum dense coding and quantum cryptography.

Here I will deal in more detail with quantum teleportation only. By this sententious title, the transfer of an unknown quantum state by means of a known entangled state and by transmission of classical information is meant. The possibility of teleportation of unknown quantum states was first noticed by Bennett *et al.*[20]

Before proceeding to explanation of the principle of quantum teleportation, let us introduce so-called Bell's states. A quantum state of two two-state particles is described in four-dimensional Hilbert space. In this space the following orthonormal basis can be chosen (Bell's states):

$$\begin{aligned} |\Psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|V\rangle_1|H\rangle_2 \pm |H\rangle_1|V\rangle_2\right), \\ |\Phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|V\rangle_1|V\rangle_2 \pm |H\rangle_1|H\rangle_2\right), \end{aligned} \tag{14}$$

here $|V\rangle_j$ and $|H\rangle_j$ ($j = 1, 2$) represent two orthogonal states of particle one or two, respectively. For clarity we may consider V to denote the vertical linear polarization of a photon and H the horizontal polarization. Each of above listed Bell's states is an entangled state of both particles. Let us now suppose to have a particle (denoted henceforth by subscript 1) in an unknown polarization state

$$|\phi_{\rm i}\rangle = \alpha|V\rangle_1 + \beta|H\rangle_1 \tag{15}$$

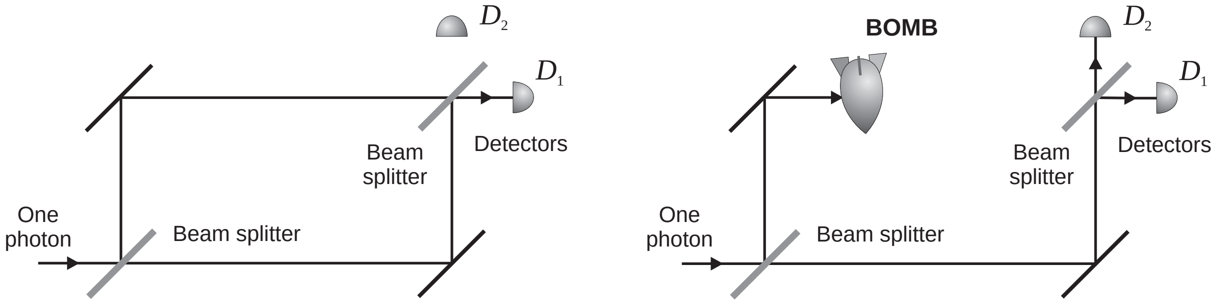

Figure 6. Mach-Zehnder interferometer with and without a "bomb".

and a pair of particles 2 and 3[§] – one of which the sender has at his disposal and the other the recipient (see Fig. 5) – with entangled polarizations, in a known state, e.g.,

$$|\phi_{\rm EPR}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_2|H\rangle_3 + |H\rangle_2|V\rangle_3). \tag{16}$$

The total state of the three-particle system is given by the direct product

$$\begin{aligned}|\phi\rangle = |\phi_{\rm i}\rangle|\phi_{\rm EPR}\rangle = \frac{1}{2}\big[&|\Psi^+\rangle\,(\alpha|V\rangle_3 + \beta|H\rangle_3) + |\Psi^-\rangle\,(\alpha|V\rangle_3 - \beta|H\rangle_3) + \\ &+ |\Phi^+\rangle\,(\alpha|H\rangle_3 + \beta|V\rangle_3) + |\Phi^-\rangle\,(\alpha|H\rangle_3 - \beta|V\rangle_3)\big].\end{aligned} \tag{17}$$

If the sender performs a quantum measurement on particles 1 and 2, which unambiguously projects his subsystem into one of four Bell's states,[21] the state of particle 3, at the side of the recipient, "collapsed" (due to the change of the total state vector caused by the measurement on particles 1 and 2) into one of the four corresponding possibilities, namely into the state $\alpha|V\rangle_3 + \beta|H\rangle_3$ in the case when $|\Psi^+\rangle$ is measured, into $\alpha|V\rangle_3 - \beta|H\rangle_3$ with $|\Psi^-\rangle$, etc. If the sender informs the recipient which Bell's state was found the recipient can – applying an appropriate unitary transformation (i.e., changing the sign and/or swapping vertical and horizontal polarizations) – reconstruct the original unknown polarization state on particle 3: $\alpha|V\rangle_3 + \beta|H\rangle_3$. So, the information transmitted has two parts: (1) Classical one referring which Bell's state was found by the sender. This part may be delivered by phone, e.g., and the speed of its transmission is limited by the speed of light. (2) Quantum part which is "transmitted instanteneously" through the medium of the entangled pair. However, it alone is not sufficient for the reconstruction of an unknown quantum state. Experiments based on two different optical implementations run in Innsbruck and in Rome.[22]

Entangled states of two two-state particles can also be employed for transmission of, theoretically, even two bits of information by means of only one of the two particles[23] – this is called quantum dense coding (in a certain sense it is an "inversion process" to quantum teleportation).

11. "INTERACTION-FREE" MEASUREMENT

To ascertain the presence of some object one can shine light on it and see it – i.e., information about things is obtained by means of some kind of interaction with them. However, Elitzur and Vaidman[24] proposed a way to check the presence of an object without the necessity to "interact" with it. The conception of interaction-free measurements is based on the wave-particle duality.

Let us imagine a Mach-Zehnder interferometer adjusted so that an incident photon always exits at detector D_1 (in this output constructive interference occurs, in the output leading to detector D_2 destructive interference does – see Fig. 6). Let us now insert an absorbing object in the upper arm of the interferometer, e.g., a bomb with an ultrasensitive triggering mechanism capable to react on even single photon. Now, if the splitting ratio of the first

[§]The numbers represent nothing else than the abbreviations for spatial parts of the states. Photons, like any other quantum particles, are indistinguishable. If we speak, e.g., about the "first" photon we just mean that "coming from the left" and the symbol $|V\rangle_1$ should be understand as "one photon in the mode corresponding to a specific spatial state of electromagnetic field with vertical polarization".

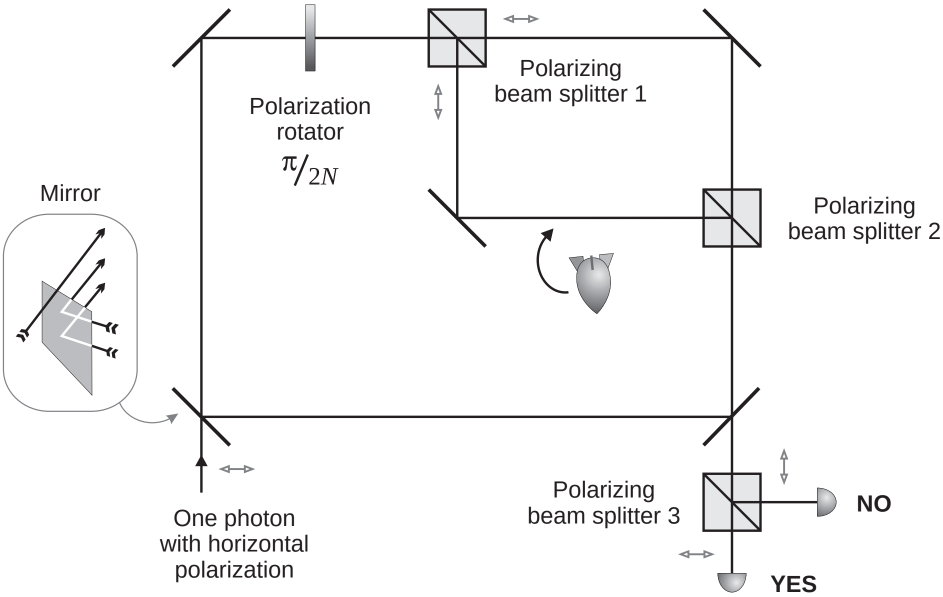

Figure 7. Interaction-free measurement.

beam splitter is 50:50, the incoming photon will go with 50 % probability to the upper arm towards the object (the bomb will explode). However, with 50 % probability it will go through the lower arm to the second beam splitter (with a splitting ratio 50:50 again) and will impinge on one of the detectors (the bomb will not explode). The probability of a click of detector D_1 is the same as the probability of a click of detector D_2 now. In other words, since the path of the photon is known there is no interference. Altogether, in 50 % of events the bomb explodes, in 25 % we obtain signal from detector D_1 yielding no information, but 25 % remain when the photon falls on detector D_2. In such a case one can conclude that an object (bomb) was certainly within the interferometer, even though the photon have not "interacted" with it. Well, such efficiency with a "bomb detection" is not very encouraging, but it is worth mentioning that any classical method would have zero efficiency. Besides, a clever trick was proposed[25] enabling us to make the percentage of successful cases arbitrarily close to 100 %. It employs an optical version of the so-called quantum Zeno effect,[25] whose principle lies in the fact that repetitionary measurements (interrupting, in short periods, unitary time evolution) keep the system, with a high probability, in unchanged state.

Let us consider the arrangement in Fig. 7. One photon with horizontal polarization enters the systems and makes N cycles along a spiral (it is provided by the geometry of configuration). In the system a polarization rotator is placed, which turns the polarization plane of the photon by an angle $90°/N$ in each cycle. There is also an "interferometer" with arms of exactly equal lengths consisting of two polarizing beam splitters separating and combining again the vertical and horizontal components of polarization. If no bomb is present, both arms are opened and the polarization of the photon is being turned step by step from horizontal to vertical one. However, if one arm is interrupted by the bomb, the path of the photon is distinguishable (if the photon goes "downward" the bomb explodes). When the photon overpasses the bomb, its polarization is not reconstructed but stays horizontal henceforth. The total probability that the bomb does not explode and the photon is registered with horizontal polarization after N cycles is

$$P_N = \left[\cos^2\left(\frac{\pi}{2N}\right)\right]^N . \tag{18}$$

It is evident that by increasing N, the value of P_N approaches unity – the "risk of explosion" can arbitrarily be minimized.

12. CONCLUSIONS

As already mentioned at the beginning, the presented selection is not and can not be complete by far. In the framework of the selected area, it would surely be interesting to mention in more detail other (already realized or only proposed) experiments – e.g., quantum beats,[27] interference of light from independent sources at the one-photon level,[28] non-locality of a single particle,[29] etc. I hope, however, that even despite its incompleteness, the contents of the previous pages approximated the current state of, at least, some branches of quantum optics and stressed its importance for the understanding of the fundamentals of quantum theory.

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