Programmable discriminator of coherent states: Experimental realization

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An optical implementation of the recently proposed unambiguous identification of coherent states is presented. Our system works as a programmable discriminator between two, in general nonorthogonal, weak coherent states. The principle of operation lies in the interference of three light beams–two program states and one unknown coherent state which can be equal to either one of the two program states. The experiment is based on fiber optics. Its results confirm theoretical predictions, and the experimental setup can be straightforwardly extended for higher numbers of program states.

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I. INTRODUCTION

Only orthogonal states of any quantum system can be discriminated perfectly and with 100% efficiency. It is possible to discriminate even nonorthogonal states but either with errors or/and with a certain number of inconclusive results. In general, the ability to discriminate quantum states is important for quantum-information transfer and processing. For example, it can serve as an efficient attack on quantum key distribution [1]. Error-prone discrimination was investigated already in the seminal work of Helstrom [2]. Later it was shown that the error-free or unambiguous discrimination of two non-orthogonal states is also possible although only in a probabilistic way [3–6]. Unambiguous discrimination of more than two nonorthogonal (but linearly independent) states was also studied [7]. The physical scheme for optimal unambiguous discrimination of coherent states was proposed by Banaszek [8]. Van Enk discussed the effectiveness of several methods for unambiguous discrimination of N symmetric coherent states using linear optics and photodetectors [9]. Many other works have dealt with the discrimination of mixed states [10–12]. Further, so-called programmable discriminators, where the set of specimen states is determined by a quantum "program," were proposed and experimentally tested [13–15]. This task can also be seen as quantum state comparison [16]; the signal state is compared with the set of specimen states. The implementation of the coherent-state comparison device was published by Andersson *et al.* [17]. Recently, Sedlák *et al.* [18] proposed experimentally feasible implementation of a scheme for unambiguous identification of coherent states with potential application to quantum database search.

In this paper, we present experimental realization of a simple version of the unambiguous identification of coherent states proposed in Ref. [18]. The unknown coherent state can be equal to either one of the two different program states. The number of program states can be increased by extension of the basic experimental scheme.

The scheme of our setup is in Fig. 1. State $|\alpha_?\rangle$ is the state to be discriminated. States $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are the program states. Our task is to find whether $|\alpha_?\rangle=|\alpha_1\rangle$, or $|\alpha_?\rangle=|\alpha_2\rangle$. As shown in Ref. [18], if the intensity transmittance of beam splitter BS_1,

$$T_1 = \frac{1}{1+T_0}, \tag{1}$$

and the transmittance of beam splitter BS_2,

$$T_2 = \frac{1-T_0}{2-T_0}, \tag{2}$$

where T_0 is the intensity transmittance of beam splitter BS_0 (we suppose that the reflectances and transmittances add to unity, $R_j+T_j=1$), then one can unambiguously identify the incoming state just by photodetection at detectors D_1 and D_2. If D_1 clicks we can conclude that $|\alpha_?\rangle=|\alpha_2\rangle$; if D_2 clicks it means that $|\alpha_?\rangle=|\alpha_1\rangle$. If neither of the detectors clicks we cannot make any conclusion about the state $|\alpha_?\rangle$–this situation corresponds to an inconclusive result. In an ideal situation the two detectors may never click in coincidence. The probability of correct identification of state $|\alpha_1\rangle$ reads

$$p_1 = 1 - \exp\left(-\eta_2 \frac{1-T_0}{2-T_0}|\alpha_1-\alpha_2|^2\right), \tag{3}$$

and of correct identification of state $|\alpha_2\rangle$

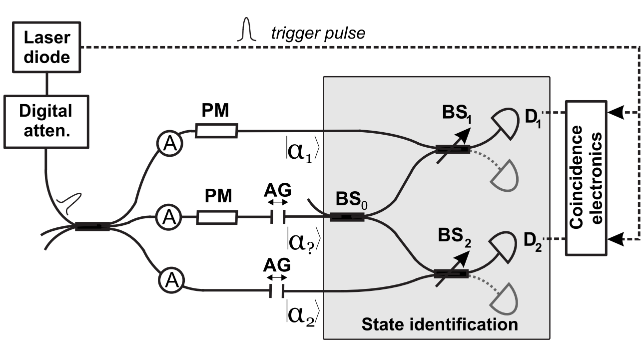

FIG. 1. The scheme of our experimental setup. A, attenuator; PM, phase modulator; AG, adjustable air gap; BS, beam splitter; D, detector.

$$p_2 = 1 - \exp\left(-\eta_1 \frac{T_0}{1+T_0}|\alpha_1 - \alpha_2|^2\right), \qquad (4)$$

where η_j denote the detection efficiencies. In this paper we assume equal prior probabilities of both coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$. In such a case the optimal choice of the splitting ratios is shown to be $T_0=1/2$, $T_1=2/3$, $T_2=1/3$ (independently of the input states) [18].

In a real setup it sometimes happens that detectors D_1 and D_2 click simultaneously. In practice, these double clicks, as well as no detections, correspond to inconclusive results because we cannot distinguish whether the unknown state was equal to state $|\alpha_1\rangle$ or to state $|\alpha_2\rangle$. All other situations, when just one of the detectors clicks, correspond to conclusive results. They include both correct and erroneous identifications of the unknown state. Ideally, erroneous identifications never occur and the probability of a conclusive result is equal to the probability of correct identification.

II. DESCRIPTION OF EXPERIMENT

The experimental setup (see Fig. 1) was built up using fiber optics. For preparation of coherent states, we used strongly attenuated pulses produced by a laser diode at wavelength 826 nm and with a length of pulse around 4 ns. Pulses were divided by a fiber coupler into three optical fibers, each corresponding to one of the states $|\alpha_1\rangle$, $|\alpha_2\rangle$, $|\alpha_?\rangle$. The amplitudes of coherent states were adjusted together by a digital attenuator located before pulse splitting and separately for individual modes by attenuators (A). The phases of the states were controlled by electro-optical phase modulators (PM).

The principle of state discrimination lies in the interference of light beams at beam splitters. The discrimination is optimal when beam splitter BS_0 is balanced; therefore a fiber coupler with fixed splitting ratio 50:50 was used. As beam splitters BS_1 and BS_2 we employed two variable-ratio couplers adjusted to desired splitting ratios. The whole setup, including both the preparation of all coherent states and the identification of the unknown state, worked basically as two interconnected Mach-Zehnder (MZ) interferometers. To accomplish the discriminating operation the visibilities of both MZ interferometers had to be maximized. This was provided by aligning polarizations and setting the same optical paths for the arms corresponding to $|\alpha_1\rangle$ and $|\alpha_?\rangle$ and those corresponding to $|\alpha_2\rangle$ and $|\alpha_?\rangle$. Before the measurement the path balance was roughly done with the help of two adjustable air gaps and precisely with phase modulators during the measurement itself.

Changes of temperature and temperature gradients cause changes of the refractive index of optical fibers and thereby a drift of phase in time. To reduce the phase-drift effect we utilized thermal isolation of the setup in a polystyrene box. Additionally an active stabilization was performed to compensate residual phase drift. Before each 3 s measurement step, phase deviations from the balanced state were monitored simultaneously in both MZ interferometers, and if necessary a proper correction was done by means of phase modulators [19].

The signal was detected by four Perkin-Elmer single-photon-counting avalanche photodiodes. Two of them, D_1 and D_2, served for both the discrimination and active stabilization, while the two others were used only for stabilization. To minimize the influence of dark counts of the detectors on a measurement, D_1 we counted coincidences between signals from detectors D_1 and D_2 and pulses that triggered the laser diode. For this purpose, coincidence electronics including time-to-amplitude convertors and single-channel analyzers were utilized. The coincidence window was set to the value of 8 ns. The mean number of dark counts in the coincidence window was approximately 4×10^{-7}, whereas the mean number of signal counts in the coincidence window in our experiment ranged from 0.002 to 0.7.

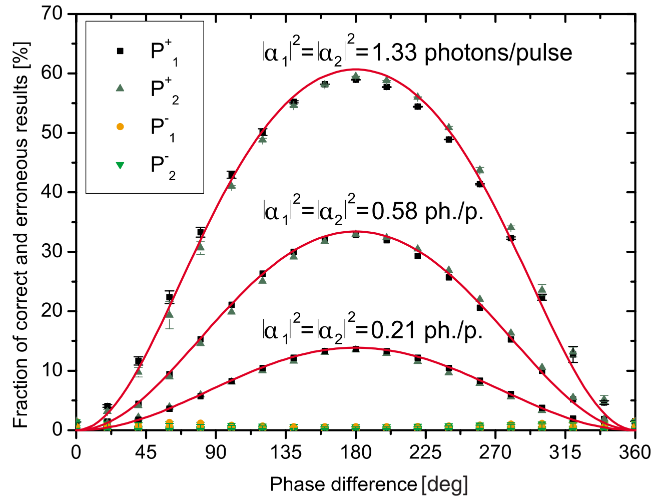

FIG. 2. (Color online) Dependence of the fraction of correct and erroneous results on the phase difference between states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ for three different intensities of states; $|\alpha_1|^2=|\alpha_2|^2$. Solid lines represent theoretical predictions for the probability of a conclusive result.

The quantum efficiences of the detectors also play an important role. They are essential for the measurement of amplitudes of coherent states and they constrain the succes probability of state identification. Efficiencies were measured by means of a cw laser diode, a well-callibrated digital attenuator, and a power meter. First we determined the power of the laser signal using the power meter. Then the laser beam was attenuated by the digital attenuator and the count rates measured by the detector were compared with the photon flux calculated from the power measured beforehand. By this measurement we obtained the efficiences of the detectors $\eta_1=\eta_2=(53\pm1)\%$.

In our experiment, we tested the state identification for various combinations of states $|\alpha_1\rangle$ and $|\alpha_2\rangle$. For each such measurement, we first set the desired intensities $|\alpha_1|^2$ and $|\alpha_2|^2$ of the program states. Then the intensity of state $|\alpha_?\rangle$ was adjusted to be equal to the intensity of either the first or the second state. By applying proper voltages on phase modulators we were able to prepare coherent states with various phases.

We measured conclusive count rates C_j^+, when the state was correctly discriminated, and C_j^-, related to erroneous detections; $j=1,2$. For example, when $j=1$ then $|\alpha_?\rangle=|\alpha_1\rangle$. C_1^+ (C_1^-) was obtained by measuring coincidence rates between detector D_2 (D_1) and trigger pulses of the laser diode minus

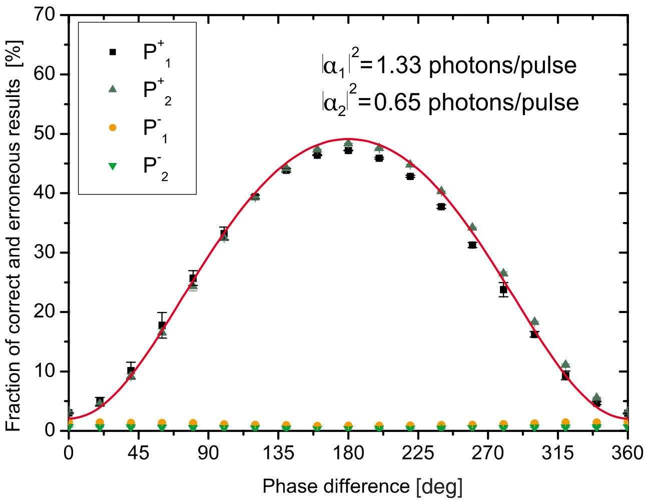

FIG. 3. (Color online) Dependence of the fraction of correct and erroneous results on the phase difference between states $|\alpha_1\rangle$ and $|\alpha_2\rangle$; $|\alpha_1|^2 \neq |\alpha_2|^2$. Solid line represents a theoretical prediction for the probability of a conclusive result.

the coincidence rates between detectors D_1 and D_2 (related to double clicks). C_2^+ and C_2^- were measured in a similar way. The fractions of correct and erroneous results read

$$P_j^+ = \frac{C_j^+}{C_{\text{tot}}}, \quad P_j^- = \frac{C_j^-}{C_{\text{tot}}} \quad (j=1,2), \tag{5}$$

respectively, where C_{tot} is the total number of laser pulses per measurement period. The fraction of conclusive results is thus $P_j = P_j^+ + P_j^-$ $(j=1,2)$.

III. RESULTS AND CONCLUSIONS

Experimental results are shown in Figs. 2–5. Each measured point was averaged from data collected during ten 3 s measurements. Error bars correspond to statistical errors from these ten measuring steps. Figures 2 and 3 display the fraction of correct and erroneous results as a function of phase difference between coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$. The theoretical curves of probabilities of a conclusive result (i.e., probabilities of correct identification) were calculated by Eqs. (3) and (4). In our case they are identical for both program states due to the equality of the efficiencies of detectors D_1 and D_2. Measured data are presented as P_j^+ and P_j^- according to Eq. (5). In the ideal case, when the visibility of interference is 100% and there are no dark counts, the probability of a conclusive result is equal to the probability of correct identification. In our setup the effect of dark counts was minimized to be negligible and visibilities were around 98%. The imperfect interference affects the quality of discrimination mainly in situations when the overlap of coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ is relatively high.

The probability of a conclusive result for the phase difference 180° between states rapidly grows with increasing intensities of states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ (see Fig. 4).

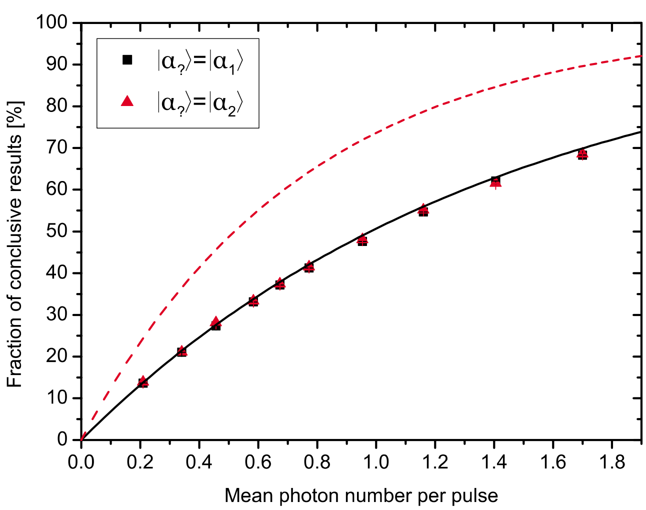

FIG. 4. (Color online) Dependence of the probability of a conclusive result on the intensity of states ($|\alpha_1|^2 = |\alpha_2|^2$; phase difference between states 180°). Solid line represents the theoretical prediction for our detectors with $\eta=53\%$. Dashed line is the theoretical limit for ideal detectors (quantum efficiency $\eta=100\%$).

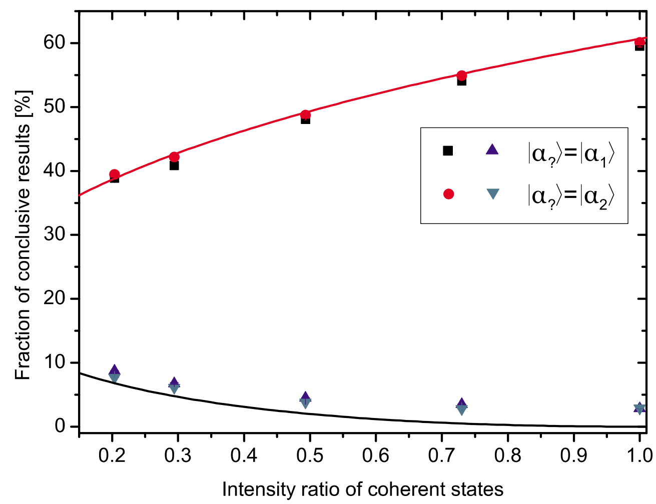

FIG. 5. (Color online) Dependence of the probability of a conclusive result on the intensity ratio of states $|\alpha_2|^2/|\alpha_1|^2$ ($|\alpha_1|^2 = 1.33$ photons/pulse). The upper line represents the theoretical prediction for the phase difference 180° between states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ and the lower line corresponds to the phase difference 0°.

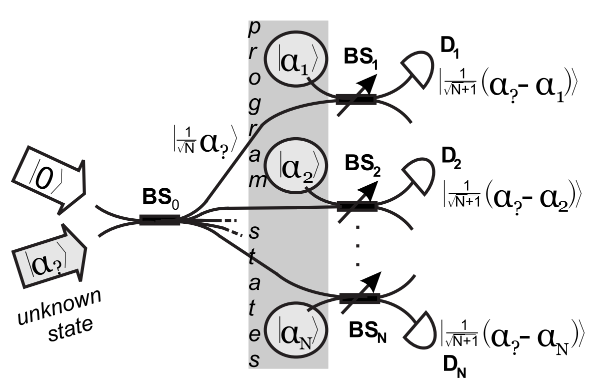

FIG. 6. Possible extension of the discrimination scheme for N program states. BS_0 equally splits an unknown state into N fibers. Splitting ratios of BS_j, $j=1,\ldots,N$ are $T_j = N/(N+1)$, $R_j = 1/(N+1)$.

Figure 5 shows the probability of a conclusive result as a function of intensity ratio $|\alpha_2|^2/|\alpha_1|^2$ where the intensity of the first state was fixed to 1.33 photons per pulse. The upper line is related to situations when the overlap of states is minimal for given intensities (phase difference 180° between the states) and the lower line corresponds to cases when the overlap is maximal (phase difference 0° between the states).

This version of coherent state identification can be straightforwardly extended for more than two specimen states (see Fig. 6). There is no experimental limitation for the extension of this scheme, and even the stabilization of more than two MZ interferometers can be performed simultaneously. However, the probability of correct identification decreases with increasing number of program states [18].

In summary, we have experimentally demonstated an unambiguous programmable discriminator of coherent states when the unknown state can be equal to one of two different specimen states. The measured values of the fraction of correct results agree well with the theoretical predictions. Our experimental setup represents a technically feasible way to implement some interesting quantum-information tasks, e.g., quantum database search [18] or an attack on quantum key distribution [9].

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[1] M. Dušek, M. Jahma, and N. Lütkenhaus, Phys. Rev. A **62**, 022306 (2000).
[2] C. W. Helstrom, *Quantum Detection and Estimation Theory* (Academic Press, New York, 1976).
[3] I. D. Ivanovic, Phys. Lett. A **123**, 257 (1987).
[4] D. Dieks, Phys. Lett. A **126**, 303 (1988).
[5] A. Peres, Phys. Lett. A **128**, 19 (1988).
[6] G. Jaeger and A. Shimony, Phys. Lett. A **197**, 83 (1995).
[7] A. Chefles and S. M. Barnett, Phys. Lett. A **250**, 223 (1998).
[8] K. Banaszek, Phys. Lett. A **253**, 12 (1999).
[9] S. J. van Enk, Phys. Rev. A **66**, 042313 (2002).
[10] J. Fiurášek and M. Ježek, Phys. Rev. A **67**, 012321 (2003).
[11] P. Raynal, N. Lütkenhaus, and S. J. van Enk, Phys. Rev. A **68**, 022308 (2003).
[12] U. Herzog and J. A. Bergou, Phys. Rev. A **70**, 022302 (2004).
[13] M. Dušek and V. Bužek, Phys. Rev. A **66**, 022112 (2002).
[14] J. Soubusta, A. Černoch, J. Fiurášek, and M. Dušek, Phys. Rev. A **69**, 052321 (2004).
[15] J. A. Bergou and M. Hillery, Phys. Rev. Lett. **94**, 160501 (2005).
[16] S. M. Barnett, A. Chefles, and I. Jex, Phys. Lett. A **307**, 189 (2003).
[17] E. Andersson, M. Curty, and I. Jex, Phys. Rev. A **74**, 022304 (2006).
[18] M. Sedlák, M. Ziman, O. Přibyla, V. Bužek, and M. Hillery, Phys. Rev. A **76**, 022326 (2007).
[19] L. Bartušková, M. Dušek, A. Černoch, J. Soubusta, and J. Fiurášek, Phys. Rev. Lett. **99**, 120505 (2007).