Experimental phase-covariant cloning of polarization states of single photons

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The experimental realization of optimal symmetric phase-covariant $1\rightarrow 2$ cloning of qubit states is presented. The qubits are represented by polarization states of photons generated by spontaneous parametric down-conversion. The experiment is based on the interference of two photons on a custom-made beam splitter with different splitting ratios for vertical and horizontal polarization components. From the measured data we have estimated the implemented cloning transformation using the maximum-likelihood method. The result shows that the realized transformation is very close to the ideal one and the map fidelity reaches 94%.

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Unknown quantum states cannot be perfectly copied [1]. The no-cloning theorem is a direct consequence of the superposition principle and linearity of quantum mechanics. The impossibility to duplicate an unknown quantum state without introducing noise is exploited by the modern quantum communication protocols and lies at the heart of the security of quantum key distribution schemes [2]. Although perfect copying is forbidden one may nevertheless copy the states in an approximate way. The optimal quantum cloning machine introduced by Bužek and Hillery in 1996 yields clones whose fidelity with respect to the input state is the maximum possible [3]. Since then, the quantum cloning has been investigated by numerous authors, see recent reviews [4,5] and references therein.

During recent years growing attention has been devoted to the experimental implementation of the various cloning machines. Optimal universal cloning of polarization states of photons has been demonstrated by several groups by exploiting the process of stimulated parametric down-conversion [6–8] or the bunching of photons which interfere on a beam splitter [9–11]. Universal cloning machines producing three copies [12] and asymmetric universal cloning machines [13] were also reported. The universal machine copies equally well all states. In many situations, however, we need to copy only a subset of the states. In particular, the phase-covariant quantum cloning machine [14,15] copies equally well all states on the equator of the Poincaré sphere, i.e., all balanced superpositions of the basis states $|0\rangle$ and $|1\rangle$, $|\psi\rangle=\frac{1}{\sqrt{2}}(|0\rangle+e^{i\phi}|1\rangle)$, where the phase ϕ is arbitrary. Due to the restriction to a smaller set of states, the $1\rightarrow 2$ phase-covariant cloner achieves slightly higher cloning fidelity $F_{\mathrm{pc}}=\frac{1}{2}(1+\frac{1}{\sqrt{2}})\approx 0.854$ than the $1\rightarrow 2$ universal cloner, whose fidelity reads $F_{\mathrm{univ}}=\frac{5}{6}\approx 0.833$.

The optimal economical phase-covariant cloning transformation requires only a single blank copy in addition to the input qubit to be cloned and reads [14]

$$|0\rangle \rightarrow |00\rangle,$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle+|10\rangle). \quad (1)$$

The optimal phase-covariant cloner represents a very efficient individual eavesdropping attack on the Bennett-Brassard 1984 (BB84) quantum key distribution protocol [16,17]. By using the asymmetric version of the cloning machine the eavesdropper can in an optimal way choose the trade-off between the information she gains on the secret key and the amount of noise that is added to the state which is sent down the communication line. It then comes as a surprise that, to the best of our knowledge, the $1\rightarrow 2$ phase covariant cloning machine has not yet been demonstrated experimentally for optical qubits. This machine has been realized in a NMR experiment [18], which, however, is not suitable for quantum communication applications where cloning of the states of single photons is desirable. Note also that Sciarrino and De Martini implemented the $1\rightarrow 3$ phase-covariant cloning of photonic qubits [19].

Here we report on the experimental realization of the optimal phase-covariant cloning transformation (1) for the polarization states of photons. Our experimental setup follows the theoretical proposal put forward in Ref. [20]. The cloning is achieved by an interference of a signal photon whose state should be cloned with an ancilla photon prepared in a fixed polarization state on a particularly tailored unbalanced beam splitter. We measure the fidelities of the two clones for a wide variety of input states and perform a maximum-likelihood estimation of the cloning operation which provides a detailed characterization of our experimental scheme. We find that due to the imperfections of our beam splitter the cloner is unbalanced and the fidelities of the two clones slightly differ. We actively compensate for this effect and symmetrize the cloner by inserting a tilted glass plate into the path of one photon.

Let us begin with a theoretical description of the cloning setup, see Fig. 1. In our scheme, the qubits are represented by polarization states of single photons. We identify the computational basis states $|0\rangle$ and $|1\rangle$ with the vertical $|V\rangle$ and horizontal $|H\rangle$ polarization states, respectively. The ancilla photon is initially vertically polarized while the signal photon can be prepared in an arbitrary state. The two photons interfere on an unbalanced beam splitter (BS) which exhibits different real amplitude transmittances t_H, t_V and reflectances r_H, r_V for the horizontal and vertical polarizations. We use the notation $R_j=r_j^2$ and $T_j=t_j^2$ for the intensity reflectances and transmittances and we have $R_j+T_j=1$ for a lossless beam

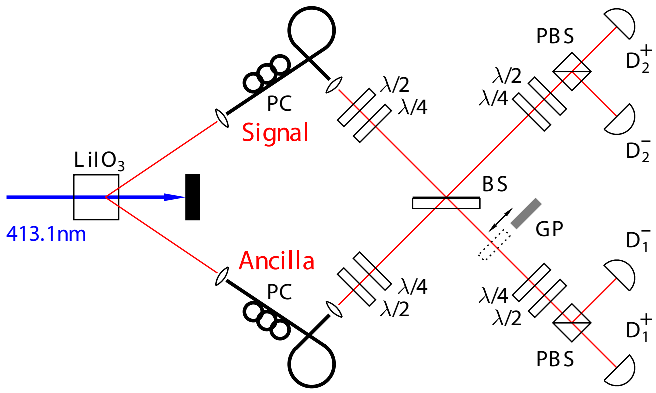

FIG. 1. (Color online) Experimental setup. For details, see text.

splitter. As shown in Ref. [20], a symmetric cloning requires $r_H = t_V$ and $t_H = -r_V$.

In the experiment, we accept only the events when there is a single photon detected in each output port of the beam splitter. The cloning transformation is thus implemented conditionally, similarly to other optical cloning experiments. The conditional transformation reads [20]

$$|V\rangle_S|V\rangle_A \to (r_V^2 - t_V^2)|VV\rangle,$$

$$|H\rangle_S|V\rangle_A \to r_V t_V(|HV\rangle + |VH\rangle). \qquad (2)$$

If r_V and $t_V = \sqrt{1 - r_V^2}$ are chosen such that $\sqrt{2} r_V t_V = r_V^2 - t_V^2$ then the device implements the optimal phase-covariant cloning transformation (1). This happens when $R_V = \frac{1}{2}\left(1 + \frac{1}{\sqrt{3}}\right) \approx 0.789$. In order to implement the cloning operation we thus need an unbalanced beam splitter with 79% reflectance for vertical polarization and 21% reflectance for horizontal polarization. For this optimal beam splitter the probability of successful cloning reads $P_{\text{succ}} = 2 R_V T_V = \frac{1}{3}$.

The scheme of our experimental setup is shown in Fig. 1. A krypton-ion cw laser (413.1 nm, 120 mW) is used to pump a 10-mm-long $LiIO_3$ nonlinear crystal (NLC) cut for frequency-degenerate (826.2 nm) type-I parametric down-conversion. The pairs of photons generated by spontaneous parametric down-conversion (SPDC) manifest tight time correlations (i.e., very exact coincidences of detection events). Both photons are coupled into single-mode optical fibers that provide spatial-mode filtration. The output beams from the fibers are set by the polarization controllers (PC) to have horizontal linear polarizations. Other polarization states are prepared by means of half-wave plates and quarter-wave plates (λ/2, λ/4)—the ancilla polarization state is fixed to a vertical linear polarization, the signal polarization state is varied. The accuracy of polarization-angle settings was better than ±1°. Then the photons enter a custom-made unbalanced beam splitter (BS) manufactured by Ekspla. The measured splitting ratios of the BS are 76:24 for vertical and 18:82 for horizontal polarization components, close to the required optimal splitting ratios of 79:21 and 21:79, respectively. This special beam splitter is a key component of our setup. Finally there are two detection blocks that can detect two chosen orthogonal polarizations. Each block consists of quarter- and half-wave plates, polarization beam splitter (PBS), and two detectors. Detectors D_1^+, D_1^-, D_2^+, D_2^- are Perkin-Elmer single-photon counting modules (employing silicon avalanche photodiodes with quantum efficiency $\eta \approx 58\%$ and dark counts about 120 s^{-1}). In each measurement the wave-plates are set such that the click of D^+ indicates projection onto the input state of the signal photon while the click of D^- heralds projection onto the orthogonal state. The signals from detectors are processed by a four-input coincidence module. Tiltable glass plate (GP) is used to compensate the imperfection of the beam splitter by implementing polarization dependent losses in one output arm. This is necessary to implement *symmetric* cloning transformation.

The cloning is successful if each photon goes by a different output arm. Therefore we measure coincidences between the detectors at two different outputs C^{++}, C^{+-}, C^{-+}, C^{--} where the first sign concerns the lower arm and the second one the upper arm, "+" means the correct result (the same as the input state), "−" means the wrong one. Fidelity of the first (second) clone thus reads

$$F_1 = \frac{C^{++} + C^{+-}}{C^{++} + C^{+-} + C^{-+} + C^{--}},$$

$$F_2 = \frac{C^{++} + C^{-+}}{C^{++} + C^{+-} + C^{-+} + C^{--}}. \qquad (3)$$

Probability of successful cloning can be determined as

$$P_{\text{succ}} = \frac{C^{++} + C^{+-} + C^{-+} + C^{--}}{C_{\text{tot}}}, \qquad (4)$$

where the total rate of events C_{tot} is obtained from the sum of all coincidence events $C_{\text{sum,dis}}$ measured with mutually delayed (i.e., distinguishable) input photons. For this measurement the signal photon is prepared in the −45° linear polarization and the ancilla remains vertically linearly polarized. The delay is realized by prolonging one input arm by 120 μm. A simple calculation reveals that $C_{\text{tot}} = C_{\text{sum,dis}}/Q$ where $Q = (T_V^2 + R_V^2 + T_V T_H + R_V R_H)/2$ is the probability that there would be a single photon in each output port of the BS. Numerically, we get $Q = 0.484$.

We prepared various input polarization states $|\psi\rangle = \cos\frac{\theta}{2}|V\rangle + \sin\frac{\theta}{2} e^{i\phi}|H\rangle$ and measured fidelities F_1, F_2 as functions of ϕ and θ. First we investigated cloning of states on the equator of the Poincaré sphere. We fixed $\theta = \pi/2$ and varied ϕ. The results are shown in Fig. 2(a). Each data point at presented plots has been derived from ten 5-s measurement periods. Symbols denote experimental data, lines represent theoretical predictions. The upper line indicates the fidelity of the optimal symmetric phase-covariant cloner $F_{\text{pc}} \approx 0.854$ while the lower line shows the fidelity of the optimal semiclassical cloning strategy based on the optimal estimation of the state [21] followed by preparation of two copies, $F_{\text{est}} = 0.75$.

We can see that the fidelities of the two clones differ by approximately 4%. This asymmetry can be attributed to the beam splitter whose reflectances somewhat differ from the ideal ones, as discussed above. The mean fidelities of the first and second clone averaged over the equator of the Poincaré

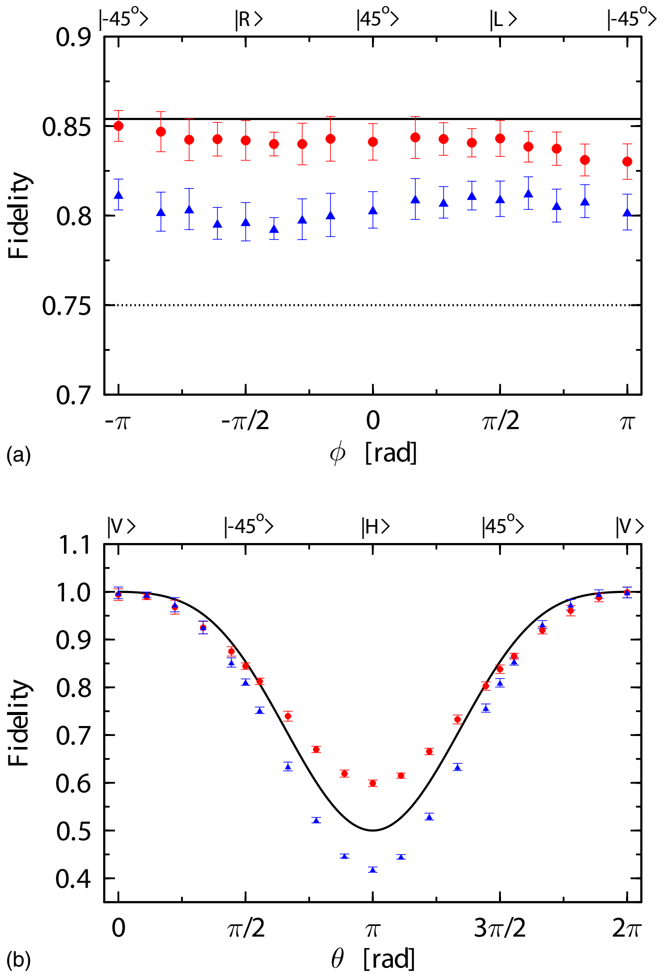

FIG. 2. (Color online) Fidelities F_1 (red circles) and F_2 (blue triangles) as functions of the input-state parameters. (a) ϕ is varied while $\theta=\pi/2$ is fixed, and (b) $\phi=0$ is fixed and θ is varied. Symbols denote experimental data, lines represent theoretical predictions.

sphere read $F_1=84.1\pm0.2\%$ and $F_2=80.4\pm0.2\%$. Both F_1 and F_2 are below the theoretical maximum F_{pc} for a symmetric cloner. This is due to several experimental imperfections, the most important ones being the imperfect overlap of the two photons on a beam splitter resulting in reduced visibility of the Hong-Ou-Mandel interference [22], the imperfect setting of the wave plates, and the random coincidences because of relatively wide coincidence window (20 ns). The fidelities are almost constant and independent of ϕ which confirms the phase covariance of the cloning device. The small modulation of F_1 and F_2 is probably caused by an imperfect preparation of the ancilla state whose deviation from $|V\rangle$ would result in the observed oscillation of fidelities.

The transformation (1) actually optimally clones all states on the northern hemisphere of the Poincaré sphere [20], i.e., all states with $|\theta|\leqslant\frac{\pi}{2}$. We have studied the cloning of states with various θ and the results for $\phi=0$ are shown in Fig. 2(b). We can see that for $\theta<\frac{\pi}{2}$ and $\theta>\frac{3\pi}{2}$ the observed fidelities are in good agreement with the theoretical values indicated by a solid line. Note that we have carried out measurements also for $\frac{\pi}{2}<\theta<\frac{3\pi}{2}$. Although the cloning machine ceases to be optimal in this region, the measured results provide a valuable characterization of the cloning machine. In particular, the asymmetry of the cloner is most clearly visible for the input state $|H\rangle$ where the two fidelities differ most significantly. We have also performed measurements similar to those shown in Fig. 2(b) but with $\phi=\frac{\pi}{2}$. The results are very similar and are not shown here.

The cloning transformation (1) is an isometry, i.e., a deterministic operation. The success probability of a conditional implementation of such operation should not depend on the input state. In our case this means that the total number of coincidences $C_{\mathrm{sum}}=C^{++}+C^{+-}+C^{-+}+C^{--}$ should be constant. In the experiment we observe that C_{sum} remains practically constant as we vary ϕ. The maximal relative change of C_{sum} when θ is varied from 0 to π is about 8%. This confirms that the transformation realized by our scheme is close to a deterministic operation (albeit implemented conditionally). The changes in C_{sum} can be attributed to the difference of the actual splitting ratios of the BS from the ideal ones.

In order to characterize the cloning transformation more completely, we employed the quantum process tomography. Using all collected experimental data we have performed a maximum likelihood (ML) estimation of the completely positive map $\mathcal{E}_{\mathrm{cl}}$ which fully specifies the cloning operation. According to Jamiolkowski-Choi isomorphism [23], any completely positive map $\mathcal{E}$ is isomorphic to a positive semidefinite operator E on the tensor product of input and output Hilbert spaces. For any (generally mixed) input state ρ_{in} the corresponding output state $\rho_{\mathrm{out}}=\mathcal{E}(\rho_{\mathrm{in}})$ can be determined as $\rho_{\mathrm{out}}=\mathrm{Tr}_{\mathrm{in}}[\rho_{\mathrm{in}}^{T}\otimes\mathbb{1}_{\mathrm{out}}E]$, where T indicates transposition with respect to a fixed basis and $\mathbb{1}_{\mathrm{out}}$ denotes an identity operator on $\mathcal{H}_{\mathrm{out}}$.

Here the input and output Hilbert spaces are Hilbert spaces of one and two qubits, respectively, hence E_{cl} is an 8×8 Hermitian positive semidefinite matrix. The ML estimation yields a transformation that is most likely to produce the observed experimental data. The advantage of this nonlinear statistical estimation method is that it guarantees the complete positivity of the estimated operation. In our case the map is not exactly trace preserving, so we have to estimate a general trace-decreasing completely positive map. We follow the procedure outlined in Ref. [24]. We extend the output Hilbert space to include a fifth virtual sink level $|S\rangle$. The rate of events associated with the detection of the state $|S\rangle$ is set to $C^{S}=C_{\mathrm{tot}}-C_{\mathrm{sum}}$. On this extended output Hilbert space we reconstruct a trace-preserving operation using the well-established iterative algorithm [24,25]. From the resulting map represented by a 10×10 matrix we extract the 8×8 submatrix which characterizes the (generally trace decreasing) cloning operation.

The results are shown in Figs. 3(a) and 3(b). The map which corresponds to the optimal cloning operation (1) reads $E_{\mathrm{opt}}=|E_{\mathrm{opt}}\rangle\langle E_{\mathrm{opt}}|$, where

$$|E_{\mathrm{opt}}\rangle=\frac{1}{\sqrt{2}}|H\rangle(|HV\rangle+|VH\rangle)+|V\rangle|VV\rangle. \qquad (5)$$

The similarity of the estimated map E_{cl} with the optimal map E_{opt} can be quantified by the map fidelity, defined as

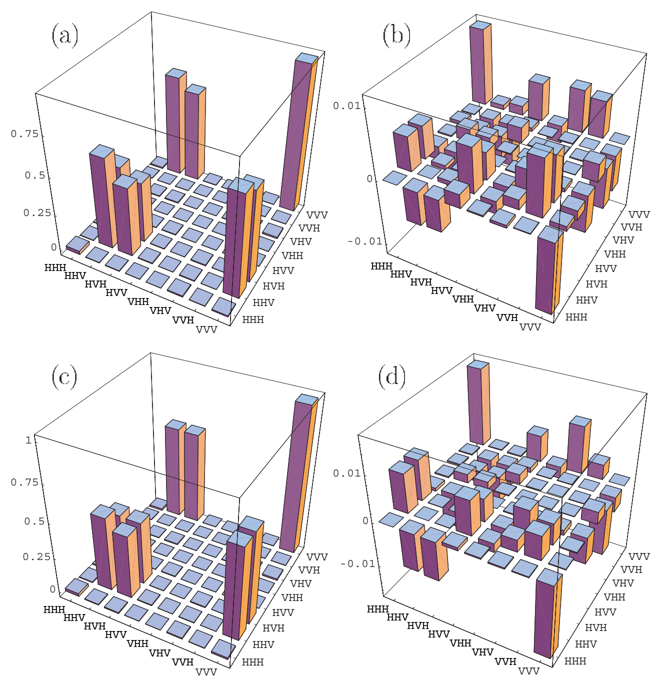

FIG. 3. (Color online) Completely positive cloning map E_{cl} estimated from the experimental data. Panels (a) and (b) show the real and imaginary parts of E_{cl} before compensation and panels (c) and (d) show the same for the map after compensation with a tilted glass plate. To facilitate the comparison, in both cases the map is normalized to Tr$[E_{\mathrm{cl}}]$=2.

$$\mathcal{F} = \frac{\langle E_{\mathrm{opt}}|E_{\mathrm{cl}}|E_{\mathrm{opt}}\rangle}{2\,\mathrm{Tr}[E_{\mathrm{cl}}]}. \tag{6}$$

For the map shown in Figs. 3(a) and 3(b) we obtain $\mathcal{F}$ =93%.

The asymmetry between the two clones is clearly revealed in the reconstructed map as the difference between the *HHV* and *HVH* matrix elements. As noted above, this is caused by the imperfections of the beam splitter, whose transmittances and reflectances do not precisely satisfy the symmetry condition $r_H = t_V$ and $t_H = -r_V$. The mapping accomplished by such a general beam splitter can be expressed as

$$|VV\rangle \to (R_V - T_V)|VV\rangle,$$

$$|HV\rangle \to r_H r_V |VH\rangle - t_H t_V |HV\rangle. \tag{7}$$

If $|r_V r_H| \neq |t_V t_H|$ then the cloner is asymmetric and produces two copies with different fidelities. To recover a symmetric copying machine, we apply an active filtering operation on one of the clones. A tilted glass plate (GP) is inserted into the path of the photon in one output port of the BS. This plate acts as a filter with different transmittances η_H and η_V for vertical and horizontal polarizations. The ratio η_H/η_V can be controlled by changing the tilt angle of the plate. With the plate present the transformation (7) changes to

$$|VV\rangle \to \eta_V (R_V - T_V)|VV\rangle,$$

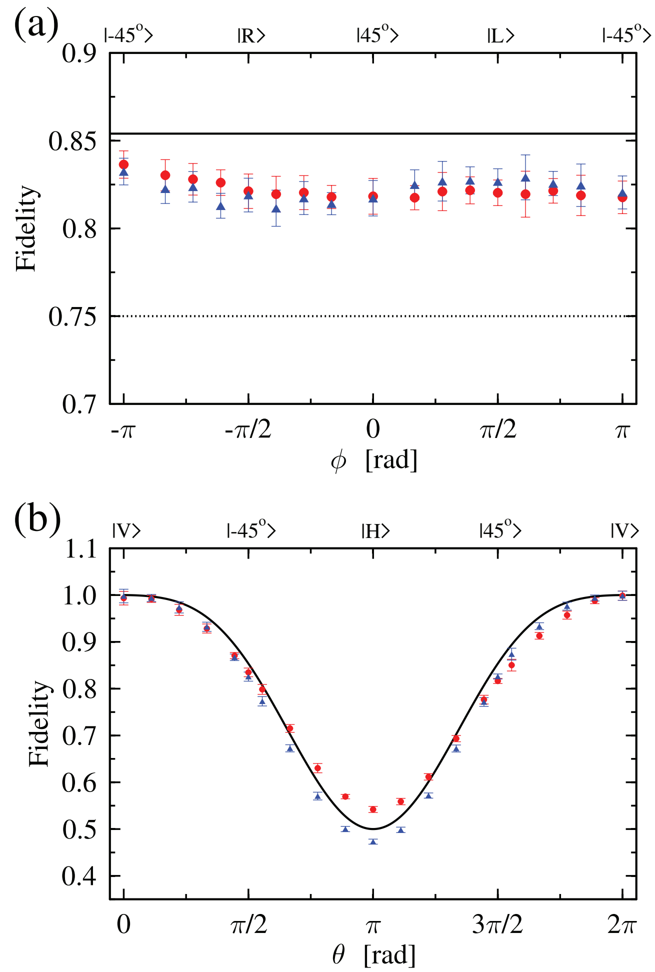

FIG. 4. (Color online) The same as Fig. 2 but with a glass plate filter inserted in the setup.

$$|HV\rangle \to \eta_V r_H r_V |VH\rangle - \eta_H t_H t_V |HV\rangle. \tag{8}$$

If we position the plate such that $\eta_H/\eta_V = |r_H r_V|/|t_H t_V|$ then we recover a symmetric cloning transformation.

The fidelities measured with the GP inserted in the setup are shown in Fig. 4. We can see that the cloner is successfully symmetrized and the fidelities of the two clones of the equatorial qubits are practically identical. The mean fidelities coincide within the measurement error, $F_1 = F_2 = 82.2 \pm 0.2\%$. The symmetrization is also clearly witnessed by Fig. 4(b) where we can see that the difference between the two fidelities for input state $|H\rangle$ is much less than before compensation, c.f., Fig. 2(b). The filtering reduced the relative variation of the total number of coincidences C_{sum} over all input states from 8% to 6% thus making the transformation very close to a (conditionally implemented) deterministic operation. The measured average success probability P_{succ} $=0.292 \pm 0.005$ is in a good agreement with the theoretical value 1/3. We have again carried out a reconstruction of the cloning map and the results are shown in Figs. 3(c) and 3(d). The filtering increased the fidelity of the map with respect to the optimal transformation E_{opt} and we find $\mathcal{F}$=94%.

In summary, we have experimentally implemented the optimal phase-covariant cloning of polarization states of single photons. The imperfections of the specifically tailored beam splitter which forms the core part of the cloning setup were compensated by a glass plate filter. In the future work we plan to investigate the possibility of optimal asymmetric phase-covariant cloning by using properly tilted glass plate filters. Another goal is to improve the parameters of the experimental setup such as to achieve for equatorial qubits cloning fidelities higher than the fidelity of the optimal universal cloning machine.

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