Bell-inequality violation with "thermal" radiation

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We demonstrate that radiation field in mixed thermal or phase-randomized coherent state can be entangled in such a way that Bell inequalities are violated. A counterintuitive result is obtained: a specific test reveals that maximal violation can be achieved with mixed states exhibiting large entropy.

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I. INTRODUCTION

In last decades, the phenomenon of entanglement between two spatially separated photons was investigated both experimentally and theoretically mainly in order to show that quantum mechanics is not a local realistic theory [1,2]. This conceptual distinction between local realistic theories and quantum mechanics is not the same as difference between entangled and separable states, and not all the entangled states must violate local realism. Entangled quantum states not admitting any local-realistic explanation are important resources in quantum communication and information processing. Beyond commonly employed entanglement in polarization degrees of freedom between two photons [3], continuous variable entangled states have been generated utilizing parametric processes [4]. Also an entanglement of the coherent states, that can be considered as the quantum analog of deterministic light waves, was studied [5]. In two-photon system, nonlocality of the polarization entanglement can be simply proved with the help of Horodecki's necessary and sufficient condition of nonlocality [6] and for multipartite case, Mermin's inequality was suggested [7]. For continuous quantum variables, nonlocality tests were proposed based on Wigner function measurements [8], construction of direct analog to the Pauli spin operators [9], or Schwinger spin operators [10] in infinite-dimensional Hilbert space.

In all the cases, the entangled states, that were used to test Bell inequalities, were usually considered to be pure states. Recently, the entanglement of mixed states has been analyzed to understand how the disorder influences on the amount of entanglement [11]. It was shown that entanglement can always arise in the interaction of an arbitrarily large system in any mixed state with a single qubit in a pure state. In addition, it was also found that a chaotic field exhibiting large entropy can nevertheless entangle the qubits that are prepared initially in a separable state.

In this paper, we consider a different scenario: A transfer of the qubit entanglement to the entanglement between thermal and vacuum states. We examine a situation, when an entangling device prepares entangled states of radiation from mixed states (thermal or phase-randomized coherent light) at the input. Similarly to the idea presented in Ref. [12], the entangling device can produce a four-mode entangled state from two mixed states and two vacuum states. It is shown, that even for very disordered states Bell inequalities can strongly be violated. If there is a narrow frequency portion of thermal radiation in the input of the entangling device then Bell inequalities are violated when the frequency of radiation is "low" and the temperature of thermal source is "high." For a phase-randomized coherent radiation the violation of Bell inequalities is even more significant. In addition, the violation can be enhanced for both the cases of radiations, if a lot of different modes are entangled with vacuum state. Thus almost the maximal Bell-inequality violation can be achieved with such thermal states exhibiting a large entropy.

The paper is organized in the following way: In Sec. II, we present the mixed entangled state and prove the entanglement by transposition criterion [13]. In Sec. III, we derive a lower bound on Clauser-Horne-Shimony-Holt-Bell (CHSH-Bell) inequality violation for the studied state. In Sec. IV, we demonstrate that the nonlocality of the generated entangled state can be enhanced as the entropy of initial thermal or phase-randomized coherent state of radiation increases. In Sec. V, the preparation of this entangled state is suggested and possible implementation in the cavity QED experiments is discussed.

II. MIXED ENTANGLED STATE

We consider two separate systems A and B that consist locally of two modes $A1,A2$ and $B1,B2$. All modes are initially unentangled. We further assume that the density matrices of these four modes are diagonal in orthonormal Fock (number-state) bases $\{|n\rangle\}$ and that the modes $A2$ and $B2$ are in vacuum states,

$$\rho_A=\sum_n p_n|n\rangle_{A1}\langle n|\otimes|0\rangle_{A2}\langle 0|,$$

$$\rho_B=\sum_m r_m|m\rangle_{B1}\langle m|\otimes|0\rangle_{B2}\langle 0|. \tag{2.1}$$

The density matrix of the total system has a factorized form $\rho_{\rm in}=\rho_A\otimes\rho_B$. Now, one can consider a conditional operation which transfers (for every $n\neq 0$ or $m\neq 0$) the factorized state: $|n\rangle_{A1}|0\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2}$, to the following entangled state:

$$|\psi_{nm}\rangle=\frac{1}{\sqrt{2}}(|n\rangle_{A1}|0\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2}-|0\rangle_{A1}|n\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2}). \tag{2.2}$$

A physical implementation of this conditional transformation is discussed in detail in Sec. V. The entangling device prepares, for each m,n, the analog of a singlet state, that was often employed to test Bell-type inequalities. Thus the initial density matrix $\rho_{\rm in}$ is transformed into the form

$$\rho_{\rm out}=N\sum_{mn} p_n r_m(1-\delta_{n0}\delta_{m0})|\psi_{nm}\rangle\langle\psi_{nm}|, \qquad (2.3)$$

where $N=[\Sigma_{nm}p_n r_m(1-\delta_{n0}\delta_{m0})]^{-1}=(1-p_0 r_0)^{-1}$. The absence of the contribution with $m=0$ and $n=0$ is the consequence of a specific postselection entangling procedure that will be explained later.

If there is at least one $n>0$ and one $m>0$ such that $p_n \neq 0$ and $r_m\neq 0$ then the state (2.3) is entangled. This can be proved in a very straightforward way using the so-called transposition criterion [13]. This criterion says that if operator ρ^{T_B}, obtained from ρ by partial transposition in subsystem B, is not positive then the state ρ is entangled. Partial transposition of

$$\rho_{\rm out}=\sum_{ijklmnst}\rho_{ijklmnst}|i_{A1}j_{A2}k_{B1}l_{B2}\rangle\langle m_{A1}n_{A2}s_{B1}t_{B2}|$$

in basis $|i_{A1}j_{A2}k_{B1}l_{B2}\rangle\equiv|i\rangle_{A1}|j\rangle_{A2}|k\rangle_{B1}|l\rangle_{B2}$ gives

$$\rho_{\rm out}^{T_B}=\sum_{ijklmnst}\rho_{ijklmnst}|i_{A1}j_{A2}s_{B1}t_{B2}\rangle\langle m_{A1}n_{A2}k_{B1}l_{B2}|.$$

Now, consider the vector

$$|\phi_{mn}\rangle=\frac{1}{\sqrt{2}}(|0\rangle_{A1}|m\rangle_{A2}|0\rangle_{B1}|n\rangle_{B2} + |m\rangle_{A1}|0\rangle_{A2}|n\rangle_{B1}|0\rangle_{B2}), \qquad (2.4)$$

where $m,n>0$ and calculate the following mean value:

$$\langle\phi_{mn}|\rho_{\rm out}^{T_B}|\phi_{mn}\rangle=-N\frac{p_m r_n}{2}. \qquad (2.5)$$

If $p_m\neq 0$ and $r_n\neq 0$ then this quantity is *negative* hence the state (2.3) is entangled. We note that the entanglement of the state (2.3) can often be "masked" by the noise of original mixed states. For example, conditional von Neumann entropy, $S(\rho_A')-S(\rho)$, is positive for many particular cases here. Nevertheless, we shall show that the entanglement is "strong" enough to violate CHSH-Bell inequality.

III. BELL-INEQUALITY VIOLATION

A natural question arises whether the entangled state (2.3) violates local realism. However, a formulation of the appropriate Bell inequalities in infinite-dimensional systems is, in general, a very complicated problem. The efficiency of nonlocality testing strongly depends on the choice of particular Bell inequalities and measured observables. In order to demonstrate the violation of Bell inequalities one needs local operations analogous to spin rotations. Since the state (2.2) lies only in a subspace of the total Hilbert space, spanned by the direct products of the basis vectors $|0\rangle_1|0\rangle_2,|n\rangle_1|0\rangle_2$, and $|0\rangle_1|n\rangle_2$ for every $n\neq 0$ in each subsystem, we can use the following unitary operation to do the job:

$$|n\rangle_1|0\rangle_2\to\cos\theta|n\rangle_1|0\rangle_2+\sin\theta|0\rangle_1|n\rangle_2 \qquad \text{for } n\neq 0,$$
$$|0\rangle_1|n\rangle_2\to-\sin\theta|n\rangle_1|0\rangle_2+\cos\theta|0\rangle_1|n\rangle_2 \qquad \text{for } n\neq 0,$$
$$|0\rangle_1|0\rangle_2\to|0\rangle_1|0\rangle_2, \qquad (3.1)$$

where the parameter θ does not depend on n.

Bell-type experiment consists of two "rotations" according to recipe (3.1), performed by two possibly spacelike separated observers, followed by realistic yes-no detection on each mode. Each such detection has only two possible outcomes (detector either fires or it does not), that can be described by projectors $|0\rangle\langle 0|$ (for "no") and $1-|0\rangle\langle 0| =\Sigma_{n=1}^{\infty}|n\rangle\langle n|$ (for "yes"). Let us assign the following values to these outcomes: $z_i=0$ if the detector (in mode i) is quiet and $z_i=1$ if it clicks. Then the results X and Y of local two-mode measurements (including "rotations") performed by the first and the second observer, respectively, can be expressed as

$$X(\theta)=z_{A1}(\theta)-z_{A2}(\theta),$$
$$Y(\theta)=z_{B1}(\theta)-z_{B2}(\theta). \qquad (3.2)$$

After the experiment is repeated many times and our two observers compare their results, the mean value of Bell operator (for CHSH inequalities) can be estimated,

$$\mathcal{B}=|C(\theta_A,\theta_B)+C(\theta_A,\theta_B')+C(\theta_A',\theta_B)-C(\theta_A',\theta_B')|, \qquad (3.3)$$

where correlation function

$$C(\theta_1,\theta_2)\equiv\sum_{j,k} X_j Y_k\, p(X_j,Y_k|\theta_A,\theta_B) \qquad (3.4)$$

(summations go over all possible results).

Every local-realistic theory [1] must fulfill the following inequality $\mathcal{B}\leqslant 2$ [2]. However, it follows from straightforward quantum-mechanical calculations that for state (2.3) the correlation function (3.4) reads

$$C(\theta_A,\theta_B)=-\cos[2(\theta_A-\theta_B)]\frac{(1-p_0)(1-r_0)}{1-p_0 r_0}. \qquad (3.5)$$

Therefore the results of the above-mentioned local measurements performed on state (2.3) can *violate* inequality $\mathcal{B}\leqslant 2$, in principle. Maximal value,

$$\mathcal{B}_{\max}=2\sqrt{2}\,\frac{(1-p_0)(1-r_0)}{1-p_0 r_0}, \qquad (3.6)$$

occurs for the angles

$$\theta_A=0,\quad \theta_A'=\frac{\pi}{4}\quad \theta_B=\frac{\pi}{8},\quad \theta_B'=-\frac{\pi}{8}. \qquad (3.7)$$

If both the mixed states have the same overlap with vacuum state $p_0 = r_0$, the condition for the violation of Bell inequality for the considered angles is given by a simple formula

$$p_0 < \frac{\sqrt{2}-1}{\sqrt{2}+1} \approx 0.1716. \tag{3.8}$$

As can be seen from Eq. (3.6), the maximum value of $\mathcal{B}$ depends on the probability of the presence of the vacuum state in the input density matrices. Thus, if the input density matrices of systems $A1$ and $B1$ do not contain the vacuum state then the maximal violation of CHSH-Bell inequality is the same as for the pure Einstein-Podolsky-Rosen maximally entangled state of two spin-$\frac{1}{2}$ particles. The mean value of Bell operator decreases as the contribution of the vacuum state increases in the mixtures. It should be emphasized that for properly chosen local measurements the violation of CHSH-Bell inequality does not depend on the randomness contained in the mixture but only on the overlaps of the vacuum state and the input density matrices.

IV. THERMAL AND PHASE-RANDOMIZED COHERENT RADIATION

There are two mixed states of special interest, namely, thermal radiation, exhibiting Bose-Einstein statistics, and phase-randomized coherent radiation, exhibiting Poissonian statistics. Let us now study the entangled states proposed in Sec. II when thermal and phase-randomized coherent states are at the input.

A single mode of thermal radiation has the density matrix

$$\rho = \sum_n \frac{\langle n \rangle^n}{(1+\langle n \rangle)^{1+n}} |n\rangle\langle n|, \tag{4.1}$$

where

$$\langle n \rangle = \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}. \tag{4.2}$$

For example, if the temperature of a radiation source (e.g., incandescent lamp) $T \approx 3000$ K and the optical frequency $\omega \approx 2.5 \times 10^{15}$ Hz, the mean value of photon number is $\langle n \rangle \approx 1.77 \times 10^{-3}$. The probability of the vacuum state in the mixture is

$$p_0 = 1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right) = \frac{1}{1+\langle n \rangle}, \tag{4.3}$$

what leads to the value $p_0 \approx 0.9982$ for the above given data. Thus in the optical region, the overlap of vacuum and thermal light is too large and the Bell-inequality violation does not occur.

The dependence of the maximal Bell-inequality violation on the parameter $\beta_i = \hbar\omega_i / k_B T_i$, $i = A, B$, of particular modes $A1, B1$ can be simply evaluated,

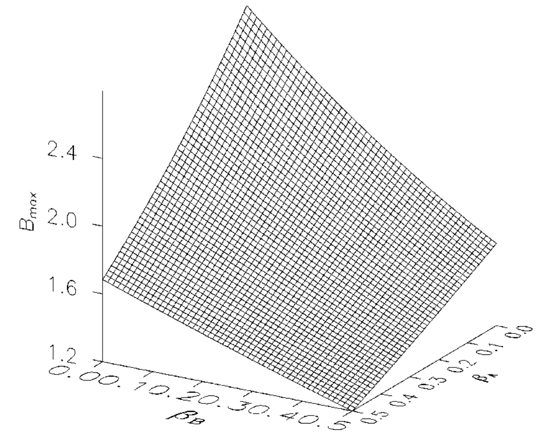

FIG. 1. The maximal violation of the CHSH-Bell inequality for thermal light as the function of parameters $\beta_A = \hbar\omega_A / k_B T_A$ and $\beta_B = \hbar\omega_B / k_B T_B$.

$$\mathcal{B}_{\max} = 2\sqrt{2}\, \frac{1}{\exp(\beta_A) + \exp(\beta_B) - 1} = 2\sqrt{2} \frac{1}{1 + \langle n \rangle_A^{-1} + \langle n \rangle_B^{-1}} \tag{4.4}$$

and it is displayed in Fig. 1. Only for very small β_A and β_B, i.e., for high temperatures and small frequencies, CHSH-Bell inequality is violated. Thus for the given temperature T of both the thermal sources the infrared component of radiation gives better results than the ultraviolet one. On the other hand, for the fixed frequency ω of both the sources the higher temperature leads to the stronger violation of Bell inequality. If the sources A and B are identical, the maximum of Bell factor

$$\mathcal{B}_{\max} = 2\sqrt{2}\, \frac{1}{2\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \tag{4.5}$$

is similar to Planck rule for mean photon number (4.2). In this case, Bell-inequality violation occurs only if the dimensionless parameter β satisfies relation $\beta < \ln[(\sqrt{2}+1)/2] \approx 0.1882$ or the mean number $\langle n \rangle$ is sufficiently large $\langle n \rangle > 2(\sqrt{2}+1) \approx 4.828$. Consequently, for the visible component of radiation the thermal sources must have an "astronomical" temperature $T > 101\,000$ K, whereas for the infrared component with $\omega \approx 5 \times 10^{13}$ Hz, temperature $T > 2021$ K is sufficient to obtain Bell-inequality violation.

Another interesting kind of mixed state is that corresponding to phase-randomized coherent light. Its density matrix can be written as

$$\rho = \sum_n \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} |n\rangle\langle n|. \tag{4.6}$$

Phase-randomized coherent radiation can be obtained from an intensity-stabilized single-mode laser with the phase uniformly distributed in the interval $\langle 0, 2\pi)$. In contrast to thermal radiation, the maximally probable state in the mixture (4.6) is not vacuum state but a Fock state $|n\rangle$, where n cor-

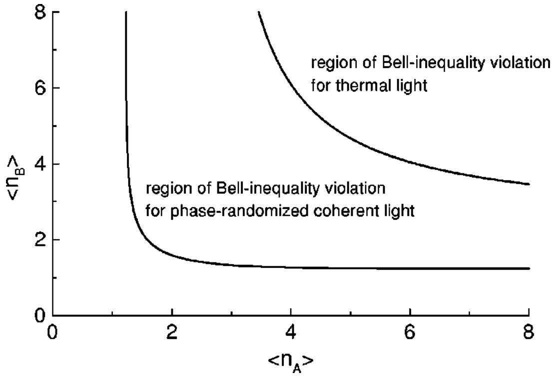

FIG. 2. The border of violation of CHSH-Bell inequality for thermal and phase-randmized coherent light in dependence on mean photon numbers $\langle n\rangle_A$ and $\langle n\rangle_B$.

responds approximately to the mean number of photons $\langle n\rangle$. Thus the overlap of phase-randomized coherent light with the vacuum state is much less than for thermal light. The probability of the vacuum state in the density matrix (4.6) is $p_0=\exp(-\langle n\rangle)$. This leads to maximal Bell-inequality violation

$$\mathcal{B}_{\max}=2\sqrt{2}\,\frac{[1-\exp(-\langle n\rangle_A)][1-\exp(-\langle n\rangle_B)]}{1-\exp[-(\langle n\rangle_A+\langle n\rangle_B)]}. \tag{4.7}$$

From Fig. 2 one can see that in the case of phase-randomized coherent light the Bell-inequality violation is achieved for less $\langle n\rangle_A$ and $\langle n\rangle_B$ than in the case of thermal light. If one considers the two identical sources of phase-randomized coherent radiation, then the Bell inequality is violated if $\langle n\rangle > \ln[(\sqrt{2}+1)/(\sqrt{2}-1)]$. At optical frequencies, lasers can generate the phase-randomized light with such a mean photon number, hence the violation can be obtained more simply than for the thermal light.

Real thermal-light sources emit to a large amount of different independent modes. The density matrix of this multimode state is given in the following form:

$$\rho=\prod_{\mu}\sum_{n_\mu=0}^{\infty}\frac{\langle n_\mu\rangle^{n_\mu}}{(1+\langle n_\mu\rangle)^{1+n_\mu}}|n_\mu\rangle\langle n_\mu|, \tag{4.8}$$

where n_μ is photon number for particular mode μ and $|n_\mu\rangle$ is the Fock state of the corresponding mode. Let us suppose that this multimode thermal state is fed to the inputs $A1$ and $B1$ and the multimode vacuum states are present in the inputs $A2$ and $B2$. The analysis presented in Sec. III may be generalized to multimode light in a straightforward way. We define the "rotations" of the multimode vacuum $|\{0\}\rangle$ and any excited multimode state $|\{n\}\rangle$ as follows:

$$|\{n\}\rangle_1|\{0\}\rangle_2\rightarrow\cos\theta|\{n\}\rangle_1|\{0\}\rangle_2+\sin\theta|\{0\}\rangle_1|\{n\}\rangle_2,$$

$$|\{0\}\rangle_1|\{n\}\rangle_2\rightarrow-\sin\theta|\{n\}\rangle_1|\{0\}\rangle_2+\cos\theta|\{0\}\rangle_1|\{n\}\rangle_2, \tag{4.9}$$

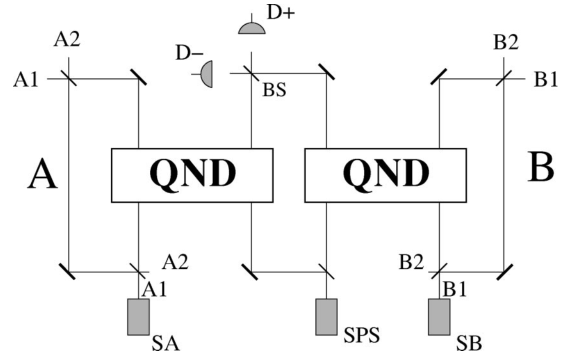

FIG. 3. Preparation device; SA and SB denote the sources of thermal (pseudothermal) radiation, SPS is a single photon source, QND is quantum nondemolition measurement performed by the Kerr interaction and D_+ and D_- are detectors.

for $\{n\}\neq\{0\}$, and $|\{0\}\rangle_1|\{0\}\rangle_2\rightarrow|\{0\}\rangle_1|\{0\}\rangle_2$ for multimode vacuum in both the modes. Detection that discriminates between the field vacuum and other states has two possible outcomes described by projectors $|\{0\}\rangle\langle\{0\}|$ and $1-|\{0\}\rangle\langle\{0\}|$. It can be shown that the maximal violation of Bell inequality exhibits the same form (3.6) as in the case of single-mode radiation, but with the following notation:

$$p_0=\prod_{\mu} p_{0,\mu}, \quad r_0=\prod_{\mu} r_{0,\mu}. \tag{4.10}$$

With increasing number of the modes of thermal radiation the effective overlap of vacuum state and such a multimode field decreases and, consequently, the maximal violation of Bell inequality is enhanced. In this way, the Bell-inequality violation can be achieved for every thermal radiation, if a sufficient number of modes is taken into account.

V. EXPERIMENTAL IMPLEMENTATION

In this section we will discuss the principle of preparation of the state (2.2) and possible experimental implementations in cavity-QED experiments. The scheme for implementation of the desired conditional transformation consists of three Mach-Zehnder (MZ) interferometers with equal-length arms [12] as is shown in Fig. 3. A single photon is fed to one input port of the central interferometer which is coupled to the left and right interferometers via nonlinear Kerr medium effectively described by the following interaction Hamiltonian:

$$H_{I,i}=\hbar\kappa a^{\dagger}a a_{i1}^{\dagger}a_{i1}. \tag{5.1}$$

Here $a^{\dagger}$ and a are the creation and annihilation operators of the mode corresponding to the left (or right) arm in the central MZ interferometer, $a_{i1}^{\dagger}$ and a_{i1}, with $i=A,B$, are the creation and annihilation operators of modes $A1$ (or $B1$), and κ is a real interaction constant.

If there is a photon in the left arm of the central MZ interferometer and the product $\kappa\tau_{\rm int}$ (where $\tau_{\rm int}$ is an effective interaction time) is set to be equal exactly to π then the described device realizes the phase shift π in the left MZ interferometer A and effectively flips the modes $A1$ and $A2$

on the output. On the other hand, if there is no photon in the left arm then the states of modes $A1$ and $A2$ remain unchanged. The described transformation is defined as follows:

$$U_A|n\rangle_{A1}|0\rangle_{A2}|1\rangle=|0\rangle_{A1}|n\rangle_{A2}|1\rangle,$$
$$U_A|n\rangle_{A1}|0\rangle_{A2}|0\rangle=|n\rangle_{A1}|0\rangle_{A2}|0\rangle, \quad (5.2)$$

where the last kets in the formulas denote the state of photon in the central MZ interferometer. The same is true about the right arm of the central MZ interferometer and modes $B1$ and $B2$. These unitary transformations U_i, $i=A,B$, can be expressed as

$$U_i=U^{\dagger}_{BS,i}U_{I,i}U_{BS,i}, \quad (5.3)$$

where $U_{BS,i}$ is the 50:50 beam-splitter transformation and $U_{I,i}$ accounts for the nonlinear interaction in Kerr medium,

$$U_{BS,i}=\exp\left[\frac{\pi}{4}(a^{\dagger}_{i2}a_{i1}-a^{\dagger}_{i1}a_{i2})\right],$$
$$U_{I,i}=\exp(i\pi a^{\dagger}aa^{\dagger}_{i1}a_{i1}). \quad (5.4)$$

So, if the photon goes through the left arm the modes $A1$ and $A2$ are flipped while the state of system B is unchanged. Completely symmetrical situation occurs, if the photon goes through the right arm.

Due to the path uncertainty of the photon in the interferometer the state of the whole system after the interaction is given by the formula

$$|\Psi\rangle=\frac{1}{\sqrt{2}}(|0\rangle|1\rangle|n\rangle_{A1}|0\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2}+i|1\rangle|0\rangle|0\rangle_{A1}|n\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2}), \quad (5.5)$$

where the kets without any subscript denote possible states of the photon inside the MZ interferometer situated in the center. Which-way information is finally erased by a beam splitter with amplitude reflectance $i/\sqrt{2}$ (the last one in the MZ interferometer) followed by two photodetectors D_+ and D_- (see Fig. 3). Depending on which one of these two detectors fires we obtain one of two possible output states of modes $A1$, $A2$, $B1$, and $B2$. Detector D_+ fires with probability $w_+=(1+\delta_{n0}\delta_{m0})/2$ and if it clicks the following state is obtained:

$$|\Psi_+\rangle=\frac{1}{\sqrt{2}}(|n\rangle_{A1}|0\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2}+|0\rangle_{A1}|n\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2}). \quad (5.6)$$

Similarly, detector D_- clicks with probability $w_-=(1-\delta_{n0}\delta_{m0})/2$ and when it fires one obtains the state

$$|\Psi_-\rangle=\frac{1}{\sqrt{2}}(|0\rangle_{A1}|n\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2}-|n\rangle_{A1}|0\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2}), \quad (5.7)$$

which is exactly the considered state (2.2).

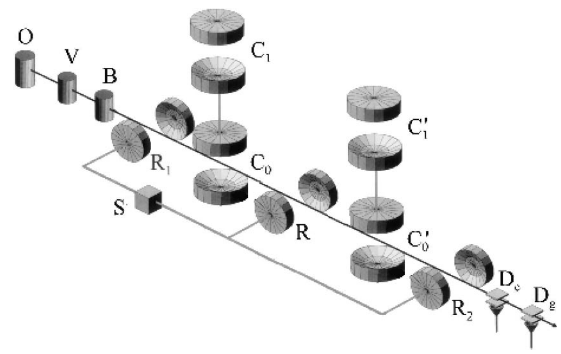

FIG. 4. Preparation device (cavity QED implementation): O, oven; V, velocity selection; B, excitation box; R,R_1,R_2, Ramsey zones; C_0,C_1,C'_0,C'_1, high-Q cavities, D_e,D_g, ionization detectors.

The crucial point of the preparation is to achieve an effective nonlinear quantum nondemolition (QND) interaction (5.1) without pronounced decoherence. QND measurements of photon number were carried out in appropriately doped optical fibers [14], however, with relatively small efficiency. Recently, it was shown that the nonlinear Kerr coupling can be enhanced using electromagnetically induced transparency in atomic vapor. A change of light pulse phase about π by the single-photon pulse is expected [15]. To realize a large cross phase modulation on a single-photon level, both cavity and free-medium regimes have been considered [16].

More efficient scheme can be implemented in double-cavity QED experiment which is depicted in Fig. 4. This experimental setup is an extension of the one previously used by Raymond *et al.* [17]. Briefly, a stream of two-level atoms serves as a set of the auxiliary systems and the electromagnetic fields in the cavities C_0,C_1 (Alice's side) and C'_0,C'_1 (Bob's side) are the systems of interest. All the cavities effectively exhibit only one field mode, particularly C_0:$A1$, C_1:$A2$, C'_0:$B1$, and C'_1:$B2$. To entangle the cavities, Alice needs to send only one atom that is detected by Bob in appropriate state.

The pair of cavities is coupled through controlled superconducting optical waveguide to perform the linear coupling (or double-side cavities can be considered). Another possibility would be to employ two-mode cavities (e.g., two polarization modes) with linear coupling between them. Thus Alice's and Bob's main interferometers are realized by the coupling between cavities, whereas Ramsey atomic interferometer is used for the auxiliary system. The duration of the experiment is typically so short that we can neglect relaxation processes in the cavities, as well as for the atoms. This approximation is realistic for experimentally achievable cavity quality factors of the order of 10^9 corresponding to photon lifetimes of the order a few milliseconds [17]. The phase manipulation of the cavity field involves three-level atoms interacting with light in the cavity. In the large detuning limit, the interaction can be effectively described by the Hamiltonian

$$H_{I2}=\hbar\kappa a^{\dagger}_1 a_1|\uparrow\rangle\langle\uparrow|,$$

with $\kappa=\Omega/\delta$, where Ω is Rabi vacuum coupling and δ is detuning between atomic and cavity frequency. As in the experiment [17], the coupling between C_0 and C_1 plays no role during the interaction (5.1), provided that the interaction time is much shorter than coupling period and it is much lower than photon lifetime in cavity.

The procedure can be performed as follows. In the first step, Alice prepares thermal states ρ_{A1} and ρ_{B1} in the cavities C_1 and C_1', whereas vacuum state is present in the cavities C_0 and C_0'. First, the coupling between the cavities C_0 and C_1 (C_0' and C_1') is switched on. After 50:50 energy exchange between C_0 and C_1, the atom effusing from an oven O (with the velocity selector in zone V) is excited into upper circular Rydberg state $|\uparrow\rangle$ in zone B. The atom is prepared, before entering C_0, in superposition $1/\sqrt{2}(|\uparrow\rangle+|\downarrow\rangle)$ by a classical microwave field applied in zone R_1. Then atom-field interaction in the cavity realizes the controlled phase-shift operation. After the interaction an additional Ramsey field R is introduced in order to establish the following operation: $|\uparrow\rangle\rightarrow|\downarrow\rangle$ and $|\downarrow\rangle\rightarrow|\uparrow\rangle$. It ensures that the flip operation (5.2) is performed either between the cavities C_0,C_1 at Alice's side or between the cavities C_0',C_1' at Bob's side. Then the atom is sent to Bob.

In the second step, another 50:50 cavity coupling follows on the both sides and the atomic states are rotated in a second classical microwave zone R_2 (performing again the same transformation as in the first Ramsey zone R_1). The atom is finally detected by field-ionization counters D_e and D_g, either in state $|\uparrow\rangle$ or in state $|\downarrow\rangle$. The measurement accuracy depends on the detector's selectivity, that is, the ability to distinguish between the two atomic states and on the velocity spread of the atomic beam. After detection, an entangled state among the four cavities is prepared.

However, the experimental verification of Bell-inequality violation in multiparticle system is a rather complicated problem. To demonstrate experimentally the effect of the overlap of the input states on the nonlocality of the output state, we can present the simplest example employing the mixed state $\rho_{A1,B1}=p_0|0\rangle\langle 0|+p_1|1\rangle\langle 1|$ in both the inputs. Then the proposed local operations (3.1) can be simply realized by the beam-splitting between modes $A1$,$A2$ and $B1$,$B2$ with different splitting ratio. It could be simply understood that Bell-inequality violation becomes stronger as the overlap of the input states becomes smaller, i.e., as modes 1 and 2 are almost in orthogonal states. The discussed thermal-radiation case in the infinite dimensional Hilbert space is an extension of this simple idea.

VI. CONCLUSION

It has been shown that two mixed states can be entangled in such a way that the entanglement of the resulting state is strong enough to violate Bell inequalities (when proper local measurements are chosen). The disorder due to the statistical nature of the density matrices of input states is irrelevant—it does not influence the violation of Bell inequality. The only parameters affecting the maximum of the mean value of Bell operator are overlaps $p_0=\langle 0|\rho_{A1}|0\rangle$ and $r_0=\langle 0|\rho_{B1}|0\rangle$. This is also the reason of a counterintuitive behavior when the entanglement increases as the input thermal state becomes more "classical" ($\beta\rightarrow 0$), whereas in the "quantum" limit ($\beta\rightarrow\infty$) the entanglement vanishes. Another counterintuitive aspect of this phenomenon appears if the multimode thermal radiation is considered. Since the overlap with multimode vacuum becomes smaller as the number of modes increases, the multimode thermal radiation can violate Bell inequality more notably, irrespective of its larger entropy. Thus this "classical-like" radiation can be strongly entangled in the ideal case and even exhibit the pronounced quantum nonlocality. Unfortunately, like other kinds of mesoscopic states, the described quantum superpositions are very sensitive to the destructive influence of decoherence and losses.

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