CAN QUANTUM NON-LOCALITY SERVE FOR INSTANTANEOUS COMMUNICATION? AN "ORTHODOX" VIEW

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Quantum mechanics is a "non-local" theory in a certain sense. The non-locality manifests itself, e.g., in correlations of results of space-like separated measurements performed on two parts of so-called *entangled* states. These correlations are "stronger" then any correlations following classical (local) conceptions. However, no measurable quantum mechanical events can break causality. No entangled state can serve for instantaneous (or superluminal) transfer of information. The proof is, at least from the view of Copenhagen interpretation, rather simple. It will be recapitulated briefly in this contribution.

1. Introduction

Quantum theory can be viewed from different philosophical positions. One can accept a (more or less positivistic) attitude, that a theory represents just a set of relations between measurable quantities, and not to care about what, e.g., the wave function is, as it cannot be measured directly. Besides, one can admit that the chance, i.e., probabilistic behavior, is inherent to microscopical phenomena and that there is no way to avoid it. Similar views were held by Niels Bohr (even if Bohr probably was not a positivist).

Such opinions are, however, very different from the "ideal of classical physics" defended by Albert Einstein. From Einstein's point of view, based on realism, a theory rather reflects behavior of real objects, whose existence is not brought into question. The classical ideal is also strictly deterministic. From this position, quantum theory appears as an uncomplete, only temporary, theory, whose stochastic character reflects just our present ignorance of some hidden parameters.

Extensive discussions between Bohr and Einstein about how to understand quantum mechanics surely contributed to the formation of the Copenhagen interpretation. Besides, in 1935 they brought Einstein, together with B. Podolsky and

N. Rosen, to the formulation of a gedanken experiment [1] employing two particles prepared in a special state to show the simultaneous existence of position and momentum, i.e., to demonstrate the overcoming of the uncertainty principle (this is why this experiment is called the EPR *paradox*). The key premises of EPR were assumptions of locality and reality:

If at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

In 1952 David Bohm showed that the EPR gedanken experiment can also be reformulated for discrete non-commuting observables, namely for different spin projections [2]. And in 1964 John Bell derived his famous inequality and showed that it is possible to arbitrate between the two above mentioned approaches (Bohr's and Einstein's) in a laboratory [3]. It opened the field for experimental tests of quantum theory vs. local realistic theories and brought to light the miraculous phenomenon of quantum "non-locality".

2. Quantum correlations

In general, a quantum system consisting of two subsystems (e.g., of two particles with spin $\frac{1}{2}$) may occur in a so called entangled state[a]. I.e., in a state whose state vector can be written in no way as the only product of any two single-particle states of the subsystems[b]. Note, however, that even the total state of the system is a pure one, neither one of the correlated subsystems is in a pure quantum state.

In case of an entangled state results of certain measurements on the both subsystems (e.g., measurements of spin projections on some axes) are mutually strongly correlated. These correlations cannot be explained by the notion of local "hidden" variables that would determine results of all potential measurements on the both subsystems and that would obtain random but correlated values at the moment when the entangled state was born. It looks like that when a quantum measurement is performed on one part of the system the other part (the other particle, e.g.) "gets to know" immediately the result no matter how far it is. Quantum measurement on one subsystem affects the state vector of the whole system.

If the quantum description of reality was uncomplete then a probability distri-

[a]Of course, more than two parts of the quantum system may be entangled [4]. However, we will be most interested in systems divided to two spatially separated entangled parts.

[b]It has no direct connection with symmetrization of the wave function of indistinguishable particles [5].

bution of different results of measurement would just reflect our ignorance of an exact actual state of the system. Then, in the moment when we would enrich our knowledge (e.g., from the result of measurement), the probability distribution would change immediately in the whole space. This is because our information on the system would change. It would be O.K. But, if the quantum state tells us everything about the system what can be told, if the wave function is really "physical", then its immediate change – an "action at distance" caused by a measurement process – seems to contradict to the causality principle[c].

However, various experimental tests which have been realized so far seem to confirm quantum mechanics with a good precision [6-8]. It implies that there is no classical theory with local hidden variables which would give, in general, the same predictions as quantum mechanics. Correlations between particles being in an entangled state are of the quantum nature. They are "stronger" than any classical ones.

3. Quantum measurement

What "really" happens at quantum measurement? Nothing detectable (like an energy, e.g.) is immediately transferred at a distance during a wave function collapse. For measurable quantities causality is not violated! The measurement results on the two subsystems may be correlated but the particular values measured are random. Two distant observers can discover the correlation only after they compare their measurement results. In this sense uncertainty protect quantum mechanics from violation of causality.

It is a difficult question what does the quantum measuring act mean in reality. In quantum mechanics, physical quantities are not straight characteristics of the system, they are, to some extent, defined by the classical measurement apparatus and their valuess may depend on the context of measurement. The essence of quantum theory is to provide a mathematical representation of states defined by practical preparation procedures together with rules for computing the probabilities of various outcomes of any test realized by a macroscopical apparatus. Collapse of the wave function is abstraction which covers up various very complex processes. Nevertheless, it works well. Of course, the part of physical system, we describe by quantum formalism, may be enlarged to involve some components of a measuring apparatus. The boundary between quantum system and classical measuring device is not fixed, it escape us as a line of horizon. Clear physical explanation of the collapse is not known. So it is difficult to answer seriously question like "how much

[c]Consider, e.g., a situation when measurements on two distant subsystems are observed from two different inertial frames. Let in the first frame the measurement on the first subsystem precede the measurement on the second one. It may happen that in the second frame succession of the measurements is inverse. Which measurement caused a collapse of the wave function?

time it takes?" etc. What is clear is that the quantum system watched become entangled with the measuring apparatus during the act of measurement so that, strictly speaking, it cannot be described by a pure state in that stage.

Next we will work with wave-function collapse as with an instantaneous process in accordance with Copenhagen interpretation of quantum mechanics.

4. Superluminal signaling

For us it is interesting, that even though the projection of a quantum state, as a consequence of measurement, takes place at once in the whole space, it cannot be utilized for an immediate information transfer at a distance. Probably the first paper dealing explicitly with the matter appears in 1978 [9], but the idea is older [10]. Later more formal discussions appeared (e.g. [11], where it has been shown that impossibility of faster-than-light signaling is a consequence of the fact that the theory is linear and that observable quantities are related to Hermitian operators).

For a simple system composed of two $\frac{1}{2}$-spin particles or of two linearly polarized photons (with correlated spins or polarizations, respectively) it can be shown, by elementary algebra of goniometric functions, that the results of measurement of projection of spin or polarization on any axis performed on either particle are quite random. However, knowing the results of measurement on the other particle of the pair one can remark correlation. No manipulation of one member of EPR pair can influence the marginal statistics of measurements on the other member [12].

The aim of this paper is to recall the important fact of impossibility of instantaneous (or superluminal) quantum communication and to present a simple "quantum mechanical exercise" proving it for an arbitrary (at least two component) quantum system.

Let us note again that our following considerations will be based on "orthodox" quantum theory. E.g., in [13] it is asserted, that a "non-orthodox" approach to the process of the wave function "collapse" (considering different durations of the process for two observers) might lead to admitting the superluminal signals. Nevertheless, considerations given in [13] was shown to be probably wrong [14]. Also some non-linear modifications to the Schrödinger equation might, perhaps, allow a superluminal signaling, see e.g. [15].

5. Conditional probabilities

Let us assume a quantum mechanical system consisting of two (spatially separated) subsystems *I* and *II*. They may be, e.g., two particles, but both subsystems may also represent rather complex entities. Let $\{|I_m\rangle\}$ and $\{|II_n\rangle\}$ are orthonormal bases in the subsystems *I* and *II*, respectively. An arbitrary pure state of the system may then be expressed in the following form

$$|\psi\rangle = \sum_{mn} \xi_{mn} |I_m\rangle |II_n\rangle, \tag{1}$$

where $\sum_{mn} |\xi_{mn}|^2 = 1$. Let further $\mathcal{O}_I$ and $\mathcal{O}_{II}$ denote sets of Hermitian operators acting only on the subsystem I or II, respectively.

The question is whether such state $|\psi\rangle$ and such operators $\mathsf{A}, \mathsf{B} \in \mathcal{O}_{II}$ and $\mathsf{C} \in \mathcal{O}_I$ exist that an observer P_I (carrying out measurement on the subsystem I) can determine, from any found eigenvalue of the operator C or at least from the statistics of the measured eigenvalues, whether an experimentalist P_{II} has measured (on the subsystem II) the observable corresponding to the operator A or to the operator B. The negative answer to this question means impossibility of an instantaneous transmission of information.

Let us suppose that the observer P_{II} performs measurement of observable $\mathsf{X} \in \mathcal{O}_{II}$ and then observer P_I measurement of observable $\mathsf{C} \in \mathcal{O}_I$. Let the operator X has eigenvalues $\{x_n\}$. For each eigenvalue there is a projection operator into corresponding eigenvector or into corresponding subspace in case of degeneration:

$$\mathsf{P}_{x_n} = \hat{\mathbf{1}}_I \otimes \sum_i |x_{ni}\rangle\langle x_{ni}|, \tag{2}$$

where $|x_{ni}\rangle$ are orthonormal eigenstates or orthonormal base vectors in subspaces corresponding to particular eigenvalues (i goes from 1 to the dimension of n-th subspace); $\hat{\mathbf{1}}_I$ denotes a unit operator acting on the subspace corresponding to the subsystem I. Since $\{|x_{ni}\rangle\}$ is a complete set these projectors represent decomposition of identity (in the whole Hilbert space):

$$\sum_n \mathsf{P}_{x_n} = \hat{\mathbf{1}}. \tag{3}$$

Similarly, if the operator C has eigenvalues $\{c_n\}$ one can introduce projectors

$$\mathsf{P}_{c_n} = \sum_i |c_{ni}\rangle\langle c_{ni}| \otimes \hat{\mathbf{1}}_{II}. \tag{4}$$

The probability that the observer P_I will obtain an eigenvalue c_k and, withal, the observer P_{II} an eigenvalue x_l, is

$$\langle\psi|\mathsf{P}_{c_k}\mathsf{P}_{x_l}|\psi\rangle = \sum_{ij} \left|\langle c_{ki}|\langle x_{lj}|\,|\psi\rangle\right|^2. \tag{5}$$

Of course, the experimentalist P_I does not know the result of the measurement of the distant observer P_{II}. Thus, the probability that P_I will measure eigenvalue c_k, when P_{II} measured some observable X, is given by the following formula (we must sum over all possible results of the measurement of P_{II})

$$p_k = \sum_l \langle\psi|\mathsf{P}_{c_k}\mathsf{P}_{x_l}|\psi\rangle = \sum_{lij} \left|\langle c_{ki}|\langle x_{lj}|\,|\psi\rangle\right|^2. \tag{6}$$

The observer P_I, seeking possibility of instantaneous signaling, would need his results depended on what the observable was measured by P_{II} (irrespective which factual eigenvalue P_{II} obtained). I.e., he would like to find whether $\mathsf{X} = \mathsf{A}$ or B. A really instantaneous transfer would demand that each single measurement of the observer P_I enabled him to discern if P_{II} had measured A or B. In other words, the set of eigenvalues of C would have been split into two disjunct subsets corresponding to two different measurements of the observer P_{II}.

However, transmission of information could work even if only the probability, that P_I measured some value c_k, depended on what P_{II} had measured (at least for some k's). In such case a transmitted message should be appropriately redundant but the information could be delivered. Possibly, the transfer of one bit could be repeated several times to obtain a statistically significant result. It would not be actually an instantaneous transfer but it could still be superluminal. Another possibility would be to use many parallel transmissions under the same conditions. Nevertheless, such situation may be described by the only (but more complex) quantum state given by Eq. (1) and the problem is reduced to the previously mentioned case.

Now, let us calculate the probabilities p_k. Using Eq. (3) one can directly see that

$$p_k = \langle\psi|\mathsf{P}_{c_k}|\psi\rangle \tag{7}$$

does not depend on P_{x_n} at all. Or, substituting explicitly from Eq. (1) into Eq. (6) one obtains

$$\begin{aligned} p_k &= \sum_{lij} \left| \sum_{mn} \xi_{mn} \langle c_{ki}|I_m\rangle\langle x_{lj}|II_n\rangle \right|^2 \\ &= \sum_{mnopi} \xi^*_{mn}\xi_{op}\langle I_m|c_{ki}\rangle\langle c_{ki}|I_o\rangle \underbrace{\left(\sum_{lj}\langle II_n|x_{lj}\rangle\langle x_{lj}|II_p\rangle\right)}_{\langle II_n|II_p\rangle=\delta_{np}} \\ &= \sum_{mnoi} \xi^*_{mn}\xi_{on}\langle I_m|c_{ki}\rangle\langle c_{ki}|I_o\rangle, \end{aligned} \tag{8}$$

where the asterisk denotes complex conjugation and δ_{np} is the Kronecker symbol. The orthogonality of $|II_n\rangle$ and completeness of $|x_{li}\rangle$ were used. The last expression is quite independent of $|x_n\rangle$ or of anything connected with X. Thus the probabilities p_k of results of measurement performed by P_I are independent of what quantity (observable) was measured by P_{II}. This must be valid for an arbitrary state $|\psi\rangle$. In other words, marginal distributions are always completely local.

6. A few additional remarks

As already mentioned, the state $|\psi\rangle$ may be rather general and both the subsystems can be considerably complex. For instance, more than two correlated particles may be present[d], a part of which is at disposal of the observer P_I and a part at disposal of P_{II} (the operators A and B may act, e.g., on different particles). Also some finite reservoirs ("environments") may be included in both subsystems, etc.

Since $|\psi\rangle$ may be an arbitrary state the proof holds not only for perfect, but also for partial entanglement.

Our observers could try to clone (to make copies of) quantum states of some particles, then to perform measurement on the copies to estimate the quantum state influenced by the measurement of the distant observer, and, consequently, to find what the distant observer has measured. A nice proof that a general quantum state cannot be perfectly cloned is given in [16]. There it is shown rather graphically that linearity of quantum mechanics forbids such replication at any quantum system. Besides, any finite quantum apparatus intended for a cloning can be involved either in one or in both subsystems considered above.

It was indicated that the violation of CP invariance could permit faster-then-light communication [17]. However, this opinion has been disproved [18]. Clearly, the proof presented here does not assume CP conservation, explicitly nor implicitly.

It is worth stressing that the proof is based on linearity of quantum mechanics (any state may be expressed as a superposition of base states) and on the fact that each observable is represented by a Hermitean operator (Hilbert space can be decomposed into a direct sum of mutually orthogonal subspaces corresponding to its eigenvalues).

7. Mixed states

The proof can easily be generalized for mixed states. A mixed state can be described by a density (or statistical) operator $\hat{\rho}$. This is a positive Hermitean operator with unit trace. Using the above defined bases, this operator takes the form

$$\hat{\rho} = \sum_{klmn} \rho_{klmn} |I_k\rangle |II_l\rangle \langle II_m| \langle I_n|, \tag{9}$$

with ρ_{klmn} being the corresponding matrix elements. The probabilities p_k introduced above for a pure state can now be rewritten as

$$\begin{aligned} p_k &= \sum_l \mathrm{Tr}\,(\mathsf{P}_{c_k}\mathsf{P}_{x_l}\,\hat{\rho}) \\ &= \sum_{lij} \langle c_{ki}|\langle x_{lj}|\,\hat{\rho}\,|x_{lj}\rangle|c_{ki}\rangle \end{aligned}$$

[d] The subscripts m and n enumerating the bases $\{|I_m\rangle\}$, $\{|II_n\rangle\}$ would then represent multi-indices.

$$= \sum_{mnli} \rho_{mlln} \, \langle I_n | c_{ki} \rangle \langle c_{ki} | I_m \rangle. \tag{10}$$

Evidently, it is independent of $|x_{nj}\rangle$ again. This probability distribution is the same for all $\mathsf{X} \in \mathcal{O}_{II}$. It depends only on the operator $\mathsf{C} \in \mathcal{O}_I$ and on the state $\hat{\rho}$.

Put it differently, if one does not know the result of the measurement of an observable X then the system after the measurement is described by a changed density operator $\hat{\rho}_X$ (von Neumann's "process 1"):

$$\hat{\rho} \longrightarrow \hat{\rho}_X = \sum_{ni} \langle x_{ni} | \hat{\rho} | x_{ni} \rangle \, |x_{ni}\rangle\langle x_{ni}|. \tag{11}$$

The reduced density operator $\hat{\rho}_I$ of the subsystem I (obtained by tracing over the subsystem II) is the same irrespective whether the change (11) has taken place or not:

$$\hat{\rho}_I = \mathrm{Tr}_{II}\, \hat{\rho} = \mathrm{Tr}_{II}\, \hat{\rho}_X. \tag{12}$$

Here $\mathrm{Tr}_{II}\, \hat{\rho} = \sum_n \langle II_n | \hat{\rho} | II_n \rangle$.

8. POVM

One can even consider more general kind of measurement. Instead of von Neumann's projectors, positive operators c_n (for observer P_I) and x_n (for observer P_{II}; n enumerates possible measurement results) may be used (see, e.g., [5]). These operators need not be projectors but they must satisfy conditions:

$$\sum_k \mathsf{c}_k = \hat{\mathbf{1}}, \qquad \sum_k \mathsf{x}_k = \hat{\mathbf{1}}. \tag{13}$$

Again, it can be readily verified that the marginal probability distribution of the results of measurement on the subsystem I,

$$p_k = \sum_l \mathrm{Tr}(\mathsf{c}_k \mathsf{x}_l \rho) = \mathrm{Tr}(\mathsf{c}_k \rho), \tag{14}$$

does not depend on what measurement was performed on the subsystem II.

9. Conclusions

Standard interpretation of quantum mechanics does not allow instantaneous transmission of information. Quantum "non-locality" appears in strong quantum correlations but causality is never violated for measurable events. Quantum correlations are a real phenomenon but they can be detected only in coincidence, i.e., comparing the results of both distant observers.

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