

Diffraction grating illuminated by partially coherent beam

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Abstract

The diffraction grating illuminated by a partially coherent radiation has been investigated in the framework of scalar wave theory. The angular distribution of light intensity behind the grating has been derived, using the formalism of correlation theory in the space-frequency domain. The source of radiation has been assumed to be quasi-homogeneous. In contrast to the coherent case, the angular dependence of intensity differs, in general, from the spectrum of radiation. However, in most of practically interesting cases this effect is negligibly small.

1 Introduction

In recent years there has been a good deal of research in the field of spectral correlation properties of partially coherent radiation. It has been predicted theoretically [1, 2, 3, 4] and then also verified experimentally [5, 6, 7, 8, 9] that the normalized spectrum of radiation is not, in general, invariant on propagation even in free space. The changes of the spectrum depend on the state of coherence of the source. The spectral features have been examined, in details, for radiation emitted by stochastic planar quasi-homogeneous sources [1, 3, 10, 11, 12] and by stochastic spherically symmetric sources [13, 14, 15]. For planar quasi-homogeneous sources a condition has been found for the normalized spectrum to be the same throughout the far zone and at the source plane: the source correlations must obey so called scaling law [1]. Also the spectral correlation properties of partially coherent radiation scattered on a random medium fluctuating in space and time have been studied [16, 17, 18]. Propagation of polychromatic partially coherent light through a linear time-invariant system [12] and through inhomogeneous medium [10] has also been studied.

Nevertheless, little attention has been paid to the process of measurement of the spectrum. Large part of spectroscopic apparatus use a diffraction grating as a main element. Diffraction of coherent waves on the grating is a well known process. But little is known about the effect of diffraction grating on propagation of partially coherent beams and on their spectral and correlation properties. In the present paper we deal with this phenomenon and we try to answer the question whether it is always the spectrum of the incident light that is measured by the use of gratings.

2 Basic concepts

First we will mention the description of a partially coherent field. We will assume a stochastic scalar field, represented by the complex analytic signal $V(\vec{\mathbf{r}}, t)$, which is statistically stationary, at least in

the wide sense. We define the correlation function of the second order

$$\Gamma(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, t_2 - t_1) = \langle V^*(\vec{\mathbf{r}}_1, t_1) V(\vec{\mathbf{r}}_2, t_2) \rangle; \quad (1)$$

here $\langle \dots \rangle$ means the ensemble average, the asterisk denotes the complex conjugate, $\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2$ are position vectors, and t_1, t_2 are arbitrary instants of time. Now we can introduce the corresponding cross-spectral density function which is the Fourier transform of the correlation function,

$$W(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \Gamma(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \tau) e^{i\omega\tau}. \quad (2)$$

This function characterizes the spatial correlation of particular frequency components. Since the field $V(\vec{\mathbf{r}}, t)$ is assumed to satisfy the homogeneous wave equation (outside sources) it can be shown that the cross-spectral density $W(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \omega)$ at arbitrary spatial point may be determined from the boundary conditions given on some surface \mathcal{D} by the following formula (see, e.g., Ref. [19])

$$W(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \omega) = \int_{\mathcal{D}} d^2 r'_1 \int_{\mathcal{D}} d^2 r'_2 K^*(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}'_1, \omega) K(\vec{\mathbf{r}}_2, \vec{\mathbf{r}}'_2, \omega) W(\vec{\mathbf{r}}'_1, \vec{\mathbf{r}}'_2, \omega), \quad (3)$$

where $K(\vec{\mathbf{r}}, \vec{\mathbf{r}}', \omega)$ represents the so called diffraction function. The spectrum of radiation $S(\vec{\mathbf{r}}, \omega) = W(\vec{\mathbf{r}}, \vec{\mathbf{r}}, \omega)$.

Let us now turn our attention to the spectrum measurement. The spectrometer works in the following manner: The diffraction grating influences the propagation of particular frequency components of incoming light beam. The detector of small angular extent is placed in sufficient distance behind the grating in a direction specified by an angle θ with the normal to the grating plane (or it is placed in the focal plane of the lens located at the distance of focal length behind the grating). The detector performs the time averaging of $|V(\vec{\mathbf{r}}, t)|^2$. If the process of fluctuations of $V(\vec{\mathbf{r}}, t)$ is ergodic the time average can be replaced by the ensemble average. It is known that in an ideal, fully coherent case the incoming waves of different frequencies are deflected in different directions.

3 Intensity in the far zone behind the grating

It can be shown, starting from Eq. (3), that the spectrum of radiation at a point $r\vec{\mathbf{s}}$ in the far zone is given by the formula [20, 21]

$$S^{(\infty)}(r\vec{\mathbf{s}}, \omega) = \left(\frac{k}{2\pi} \right)^2 \frac{\cos^2 \theta}{r^2} \mathcal{F}^4 \left[W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) \right] (k\mathbf{s}_{\perp}, k\mathbf{s}_{\perp}), \quad (4)$$

where $W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)$ is the cross-spectral density on the initial “source” plane ($\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are two-dimensional vectors^{#1}, $\vec{\mathbf{s}}$ is a unit vector and \mathbf{s}_{\perp} is its two-dimensional projection to the “source” plane, θ is the angle that $\vec{\mathbf{s}}$ makes with the axis perpendicular to the “source” plane ($\cos \theta = \sqrt{1 - |\mathbf{s}_{\perp}|^2}$), $k = \omega/c$ with c being the speed of light in vacuo, and \mathcal{F}^4 denotes the four-dimensional Fourier transform,

$$\mathcal{F}^4 \left[W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) \right] (\mathbf{f}_1, \mathbf{f}_2) = \int_{\mathbb{R}^2} d^2 \rho_1 \int_{\mathbb{R}^2} d^2 \rho_2 W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) e^{i\mathbf{f}_1 \cdot \boldsymbol{\rho}_1} e^{-i\mathbf{f}_2 \cdot \boldsymbol{\rho}_2}. \quad (5)$$

The total signal from a small (infinitesimal) detector located at the direction $\vec{\mathbf{s}}$ is proportional to the integral

$$I(\vec{\mathbf{s}}) = r^2 \int_0^{\infty} d\omega \Phi(\omega) S^{(\infty)}(r\vec{\mathbf{s}}, \omega), \quad (6)$$

where $\Phi(\omega)$ represents the spectral sensitivity of detector. The factor r^2 is added to eliminate an unsubstantial dependence on the distance r .

Let us consider an arrangement shown in Fig. 1. We will assume that immediately in front of the grating there is a planar quasi-homogeneous secondary source generating beam-like field. The cross-spectral density across the source plane can be written in the form [3, 20, 21]

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \left[S^{(0)}(\boldsymbol{\rho}_1, \omega) \right]^{1/2} \left[S^{(0)}(\boldsymbol{\rho}_2, \omega) \right]^{1/2} \mu^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega), \quad (7)$$

where $S^{(0)}(\boldsymbol{\rho}, \omega)$ is the source spectrum and $\mu^{(0)}(\boldsymbol{\rho}', \omega)$ denotes the degree of spectral coherence which depends on $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ only through the difference $\boldsymbol{\rho}' = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1$. It is assumed that $S^{(0)}(\boldsymbol{\rho}, \omega)$ vary much more slowly with $\boldsymbol{\rho}$ than $\mu^{(0)}(\boldsymbol{\rho}', \omega)$ with $\boldsymbol{\rho}'$ so that we can approximate Eq. (7) by the formula

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S^{(0)}[(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2, \omega] \mu^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega). \quad (8)$$

We chose a simple grating with a complex amplitude transmittance described by the function

$$\mathcal{M}(\boldsymbol{\rho}) = \exp\left(i \frac{2\pi}{\Lambda} \boldsymbol{\rho} \cdot \mathbf{A}\right), \quad (9)$$

where \mathbf{A} is the grating period ($\Lambda = |\mathbf{A}|$). Then the cross-spectral density immediately behind the grating takes the form

$$W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \mathcal{M}^*(\boldsymbol{\rho}_1) \mathcal{M}(\boldsymbol{\rho}_2) W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega). \quad (10)$$

On substituting into Eq. (4) and then into Eq. (6) we finally obtain the following expression for the detected intensity at the direction $\vec{\mathbf{s}}$:

$$I(\vec{\mathbf{s}}) = \int_0^\infty d\omega \Phi(\omega) \left(\frac{k}{2\pi}\right)^2 \cos^2 \theta \tilde{S}^{(0)}(\mathbf{0}, \omega) \tilde{\mu}^{(0)}\left(k\mathbf{s}_\perp - \frac{2\pi}{\Lambda} \mathbf{A}, \omega\right). \quad (11)$$

Here tilde denotes the two-dimensional Fourier transform, viz.,

$$\tilde{S}^{(0)}(\mathbf{f}, \omega) = \int_{\mathbb{R}^2} d^2 \rho S^{(0)}(\boldsymbol{\rho}, \omega) e^{-i \mathbf{f} \cdot \boldsymbol{\rho}}, \quad (12)$$

$$\tilde{\mu}^{(0)}(\mathbf{f}, \omega) = \int_{\mathbb{R}^2} d^2 \rho \mu^{(0)}(\boldsymbol{\rho}, \omega) e^{-i \mathbf{f} \cdot \boldsymbol{\rho}}. \quad (13)$$

Let us remind that in order for a quasi-homogeneous secondary source to generate beam that propagates close to the axis perpendicular to the source plane (z -axis), $|\tilde{\mu}^{(0)}(k\mathbf{s}_\perp, \omega)|$ must be negligible except when $|\mathbf{s}_\perp| \ll 1$; i.e., $|\tilde{\mu}^{(0)}(\mathbf{f}, \omega)|$ must be negligible unless $|\mathbf{f}| \ll k$ [3]. Nevertheless, the nonzero angular divergence is inherent to any partially coherent beam.

We can set, without loss of generality, $\mathbf{A} = (\Lambda, 0)$; i.e., we can choose such coordinate system that the grooves are parallel to the y -axes (see Fig. 1). Let us further suppose that the detector, though small, has a finite extent and can receive the entire optical intensity spread in the y -direction (the beam is assumed to have a very small angular divergence). This enables us to confine the considerations to only one dimension and to get rid of the delta-function in the coherent limit. Mathematically it means that $I(\vec{\mathbf{s}})$ is integrated over s_y . For simplicity let also $\tilde{S}(\mathbf{0}, \omega) = \mathcal{S}(\omega)D$ where D is an effective area of the source. We will also assume that both Λ and s_x are positive.

4 Coherent limit

It is clear from Eq. (13) that in the coherent limit and for normal incidence

$$\tilde{\mu}^{(0)}(\mathbf{f}, \omega) = (2\pi)^2 \delta(\mathbf{f}).$$

Thus

$$\mathcal{I}(s_x) = \int_{-1}^1 ds_y I(\vec{\mathbf{s}}) = \frac{2\pi c}{\Lambda} D \cos^2 \theta \frac{1}{s_x^2} \Phi\left(\frac{2\pi c}{\Lambda} \frac{1}{s_x}\right) \mathcal{S}\left(\frac{2\pi c}{\Lambda} \frac{1}{s_x}\right), \quad (14)$$

where $s_x = \sin \theta$, c being the speed of light in vacuo. Eq. (14) shows a well known fact that measuring intensity at different directions behind the grating we can determine the spectrum of incident coherent radiation (using, of course, detector with known spectral sensitivity).

5 Partially coherent beam

In the general case when the grating is illuminated by a partially coherent beam the measured quantity $\mathcal{I}(s_x)$ is expressed by the formula

$$\mathcal{I}(s_x) = \int_0^\infty d\omega \Phi(\omega) \frac{k}{2\pi} \cos^2 \theta \mathcal{S}(\omega) D \hat{\mu} \left(ks_x - \frac{2\pi}{\Lambda}, \omega \right), \quad (15)$$

where

$$\hat{\mu}(f_x, \omega) = \int_{-\infty}^\infty dx \mu(x, 0; \omega) e^{-if_x x} = \frac{1}{2\pi} \int_{-\infty}^\infty df_y \tilde{\mu}(f_x, f_y; \omega) \approx \frac{k}{2\pi} \int_{-1}^1 ds_y \tilde{\mu}(f_x, ks_y; \omega). \quad (16)$$

The last approximation holds when $\tilde{\mu}(\mathbf{f}, \omega)$ is not negligible only for $|\mathbf{f}| \ll k$. Realizing this constraint we could also presume that $\cos \theta$ is determined only by s_x so that we can not take it into account in integration over s_y . The quantity $\hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right)$, as a function of frequency ω , may be, in general, very different from the delta-function.

The question arises under what conditions can the relation (15) be approximated by Eq. (14), i.e., when the angular spread of each frequency component may be neglected and the quantity $\hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right)$ can be replaced by $2\pi \delta \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda} \right)$.

Let us suppose that the spatial Fourier transform of the degree of spectral coherence $\tilde{\mu}(\mathbf{f}, \omega) = 0$ for $|\mathbf{f}| > \alpha k$, where $0 < \alpha \ll 1$, and for each ω . This means that the beam has a very small divergence. Let also $s_x > \alpha$. Then the function $\hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right)$ will be nonzero only for frequencies ω from the following range

$$\frac{2\pi c}{\Lambda} (s_x + \alpha)^{-1} \leq \omega \leq \frac{2\pi c}{\Lambda} (s_x - \alpha)^{-1}. \quad (17)$$

We can introduce the ‘‘width’’ of $\hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right)$:

$$\Delta\omega = \frac{2\pi c}{\Lambda} \frac{2\alpha}{s_x^2 - \alpha^2}.$$

Let us set

$$\omega_0 = \frac{2\pi c}{\Lambda} \frac{1}{s_x}.$$

If $\alpha \ll s_x$ then

$$\Delta\omega \approx \omega_0 \frac{2\alpha}{s_x} = \frac{\Lambda\alpha}{\pi c} \omega_0^2 \quad (18)$$

and it is clear that $\Delta\omega \ll \omega_0$. If, in addition, the spectrum of radiation $\mathcal{S}(\omega)$ varies slowly in comparison with $\hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right)$ on the frequency range specified by inequalities (17) and if similar restriction holds also for the spectral sensitivity of detector $\Phi(\omega)$ we can express Eq. (15) in the approximate form

$$\mathcal{I}(s_x) \approx \Phi(\omega_0) \frac{\omega_0}{2\pi c} \cos^2 \theta \mathcal{S}(\omega_0) D \int_0^\infty d\omega \hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right). \quad (19)$$

Otherwise, when the spectral line is too narrow it will appear broadened.

Note that the key condition $\alpha \ll s_x$ is usually very well satisfied in real optical spectrometers. For example, for grating with 1000 lines/mm ($\Lambda = 10^{-6}$ m) the angle corresponding in the first order to the wavelength $\lambda = 550$ nm ($\omega \approx 3.4 \cdot 10^{15}$ s $^{-1}$) is about 33° , i.e., $s_x \approx 0.55$.

However, the problem remains regarding the value of the last integral in Eq. (19). In order that Eq. (19) had the same form as Eq. (14) corresponding to the coherent case, the last integral in Eq. (19) should be equal $2\pi c/s_x$. It is satisfied, e.g., when the degree of spectral coherence at the source plane is independent on frequency, at least in interval given by Eq. (17).

If the degree of spectral coherence μ varies with frequency so slowly that in interval (17) the function $\hat{\mu}$ can be approximated by the first two terms of its power expansion with respect to variable ω then

$$\int_0^\infty d\omega' \hat{\mu} \left(\frac{\omega'}{c} s_x - \frac{2\pi}{\Lambda}, \omega' \right)$$

$$\begin{aligned}
&\approx \int_0^\infty d\omega' \left\{ \hat{\mu} \left(\frac{\omega'}{c} s_x - \frac{2\pi}{\Lambda}, \omega_0 \right) + \frac{\partial}{\partial \omega} \left[\hat{\mu} \left(\frac{\omega'}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right) \right]_{\omega=\omega_0} (\omega' - \omega_0) \right\} \\
&= \frac{2\pi c}{s_x} + \frac{2\pi c^2}{s_x^2} \int_{-\alpha k_0}^{\alpha k_0} df_x \frac{\partial}{\partial \omega} [\hat{\mu}(f_x, \omega)]_{\omega=\omega_0} f_x,
\end{aligned} \tag{20}$$

where $k_0 = \omega_0/c$. Here we have used the relation

$$\int_{-\infty}^{\infty} df_x \hat{\mu}(f_x, \omega) = 2\pi \quad \text{for each } \omega, \tag{21}$$

which follows from the obvious fact that $\mu(\mathbf{0}, \omega) = 1$. The function $\hat{\mu}(f_x, \omega)$ is still assumed to be zero for $|f_x| > \alpha k$ and it is still presumed that $s_x \gg \alpha$. If $\mu(\rho'_x, \rho'_y; \omega)$ has, in addition, the following property of symmetry,

$$\mu(\rho'_x, 0; \omega) = \mu(-\rho'_x, 0; \omega) \tag{22}$$

for all ρ'_x and ω , the function $\hat{\mu}(f_x, \omega)$ is also symmetric and the last integral in Eq. (20) has zero value. Eq. (19) can be replaced by Eq. (14).

If the degree of spectral coherence of the secondary source obeys the scaling law^{#2}, i.e., if $\hat{\mu}(f_x, \omega) = k^{-1} \hat{h}(f_x/k)$ (the circumflex denotes the one-dimensional Fourier transform), then

$$\int_0^\infty d\omega \hat{\mu} \left(\frac{\omega}{c} s_x - \frac{2\pi}{\Lambda}, \omega \right) = \int_0^\infty d\omega \frac{c}{\omega} \hat{h} \left(s_x - \frac{2\pi c}{\Lambda \omega} \right) = c \int_{-\alpha}^{\alpha} dq \frac{1}{s_x - q} \hat{h}(q). \tag{23}$$

Here we have used the fact, that $\hat{h}(q) = 0$ for $|q| > \alpha$, which follows from the assumption that $\hat{\mu}(f_x, \omega) = 0$ for $|f_x| > k\alpha$. Since $s_x \gg \alpha$ the last expression in Eq. (23) can be approximated by the formula

$$\frac{c}{s_x} \int_{-\alpha}^{\alpha} dq \hat{h}(q) = \frac{2\pi c}{s_x}, \tag{24}$$

where the relation (21) has been employed again. Thus the grating “gives” actually the spectrum like in the coherent limit. This case is physically important because most common thermal laboratory sources obey the scaling law [1].

6 General scalar approach

We will now return to Eqs. (4), (6) and (7) and will describe the case of a general grating and the effect of finite aperture. Let the cross-spectral density at the initial plane have the form

$$W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = X^*(\boldsymbol{\rho}_1, \omega) X(\boldsymbol{\rho}_2, \omega) \mu(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega), \tag{25}$$

where $\mu(\boldsymbol{\rho}', \omega)$ is the degree of spectral coherence of a secondary source and

$$X(\boldsymbol{\rho}, \omega) = \mathcal{M}(\boldsymbol{\rho}) \mathcal{A}(\boldsymbol{\rho}) [\mathcal{S}(\omega)]^{1/2}. \tag{26}$$

Here $\mathcal{A}(\boldsymbol{\rho})$ involves the aperture function and the normalized distribution of the square-root of light intensity at the initial plane. $\mathcal{M}(\boldsymbol{\rho})$ is the complex amplitude transmittance of the diffraction grating; it is a periodic function which may be represented in the form

$$\mathcal{M}(\boldsymbol{\rho}) = \sum_{n=-\infty}^{\infty} \gamma_n \exp \left[in \frac{2\pi}{\Lambda^2} \mathbf{A} \cdot \boldsymbol{\rho} \right], \tag{27}$$

where γ_n are constants. The symbol $\mathcal{S}(\omega)$ denotes the source spectrum.

Using Eq. (5) we obtain the formula

$$\begin{aligned}
\mathcal{F}^4 \left[W^{(\text{init})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) \right] (\mathbf{f}_1, \mathbf{f}_2) &= (2\pi)^2 \tilde{X}^*(\mathbf{f}_1, \omega) \tilde{X}(\mathbf{f}_2, \omega) \tilde{\mu}(\mathbf{f}_1, \omega) \delta(\mathbf{f}_2 - \mathbf{f}_1) \\
&= \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} d^2 \xi \tilde{X}^*(\boldsymbol{\xi} + \mathbf{f}_1 - \mathbf{f}_2, \omega) \tilde{X}(\boldsymbol{\xi}, \omega) \tilde{\mu}(\mathbf{f}_2 - \boldsymbol{\xi}, \omega),
\end{aligned} \tag{28}$$

where $\overset{4}{*}$ denotes the four-dimensional convolution and

$$\tilde{X}(\mathbf{f}, \omega) = \tilde{\mathcal{M}}(\mathbf{f}) \overset{2}{*} \tilde{\mathcal{A}}(\mathbf{f}) [\mathcal{S}(\omega)]^{1/2}. \quad (29)$$

The tilde denotes the two-dimensional Fourier transform [as in Eqs. (12) and (13)] and $\overset{2}{*}$ denotes two-dimensional convolution. Evidently,

$$\tilde{\mathcal{M}}(\mathbf{f}) = (2\pi)^2 \sum_{n=-\infty}^{\infty} \gamma_n \delta\left(\mathbf{f} - n \frac{2\pi}{\Lambda^2} \mathbf{A}\right) \quad (30)$$

and than

$$\tilde{X}(\mathbf{f}, \omega) = [\mathcal{S}(\omega)]^{1/2} \sum_{n=-\infty}^{\infty} \gamma_n \tilde{\mathcal{A}}\left(\mathbf{f} - n \frac{2\pi}{\Lambda^2} \mathbf{A}\right). \quad (31)$$

On substituting from Eq. (31) into Eq. (28) and using Eqs. (4) and (6) we finally obtain the following expression for the detected intensity at the direction \vec{s} :

$$I(\vec{s}) = \frac{\cos^2 \theta}{(2\pi)^4} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \gamma_m^* \gamma_n \int_{\mathbb{R}^2} d^2 f \underbrace{\tilde{\mathcal{A}}^*\left(\mathbf{f} - m \frac{2\pi}{\Lambda^2} \mathbf{A}\right) \tilde{\mathcal{A}}\left(\mathbf{f} - n \frac{2\pi}{\Lambda^2} \mathbf{A}\right)}_1 \times \underbrace{\int_0^{\infty} d\omega \mathcal{S}(\omega) \Phi(\omega) \frac{\omega^2}{c^2} \tilde{\mu}\left(\frac{\omega}{c} \mathbf{s}_{\perp} - \mathbf{f}, \omega\right)}_2. \quad (32)$$

Here the summation is taken over all orders of the diffraction grating. The term designated as “1”, together with the integration over the variable \mathbf{f} , represent the effect of finite aperture. The term “2” corresponds to the elementary case which has been discussed in the previous sections.

7 Conclusions

We have described the effect of a diffraction grating illuminated by partially coherent radiation in the scalar approximation. We have shown that the angular distribution of the intensity of radiation in the far zone behind the grating is not, in general, determined by the spectrum of radiation in the grating plane alone but that it also depends on the coherence properties of the radiation. We have found conditions, however, under which this effect is negligible. These conditions are satisfied in many common practical situations.

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Footnotes

- #¹ Three-dimensional vectors are typed by boldface letters with arrows, two-dimensional vectors by boldface letters without arrows.
- #² $\mu(\boldsymbol{\rho}, \omega) = h(k\boldsymbol{\rho})$ and than $\tilde{\mu}(\mathbf{f}, \omega) = k^{-2}\tilde{h}(\mathbf{f}/k)$ where h is any “reasonable” function; $h(\mathbf{0}) = 1$. See Ref. [1].

Figure

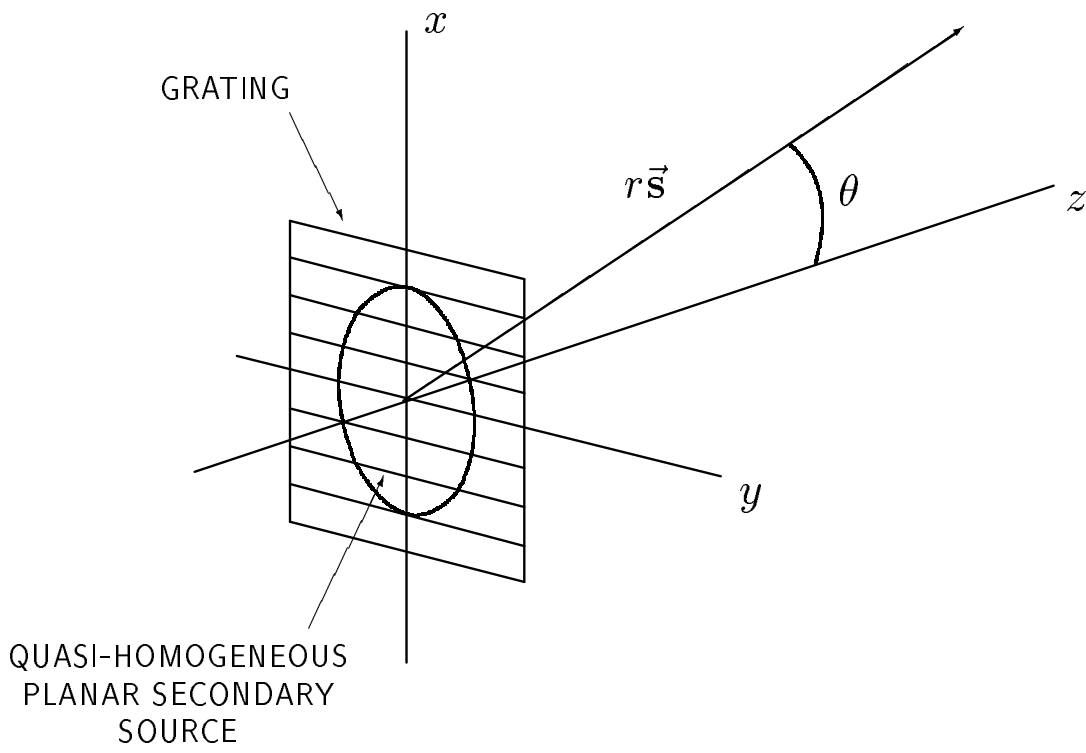


Fig. 1 Illustrating the notation.