“Nonlocal” interference effects in frequency domain

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Abstract

It is a well known fact that interference is observable as a variation of intensity only when the path difference between two arms of an interferometer is shorter than the coherence length of the light. Nevertheless, interference effects do not vanish in such case, but they manifest themselves as a modulation of the spectrum. It is also known that the photon pairs produced by spontaneous parametric down-conversion show energy correlation (entanglement). A quantum measurement on one photon of the entangled pair affects considerably the whole system due to the “collapse” of the wave function. The correlation in entangled states is purely a quantum effect. It will be shown that the interference in frequency domain and the “nonlocality” of quantum mechanics may appear simultaneously. An experiment is proposed which should demonstrate that if a filter providing spectral selection is placed in the route of one photon of the entangled pair and the photon is detected behind it, then interference appears in the (distant) Mach-Zehnder interferometer placed in the route of the other photon of the pair even if the optical path difference through the interferometer exceeds the coherence length of the light and if the spectra of these two photons do not overlap. The effect described represents a very graphical illustration of strong frequency correlation of the considered two-photon entangled state.

1 Introduction

In the recent past there was a good deal of research concerning the experimental tests of quantum nonlocality on the basis of quantum optics. Various experiments were performed demonstrating violation of Bell’s inequalities (and other classical inequalities) and showing evidence of quantum entanglement [1]. In these experiments, both the states exhibiting nonlocal spin or polarization correlations [2] and entangled states with frequency and momentum correlations [3] were used. The latter usually in the form of two entangled photons produced by spontaneous parametric down-conversion. These experiments play an important role in physics. They deepen the understanding of the fundamental features of quantum mechanics and they indicate the impossibility to replace quantum mechanics by a classical theory with local hidden variables [4]. Besides, nonlocal phenomena and related experimental techniques find an interesting practical application in quantum cryptography [5].

Many experiments are based on various artfully chosen interference effects of different orders. However, if the path difference between two arms of an interferometer is greater than the longitudinal coherence length of the light, the interference (of the 2nd order) cannot be observed as a variation of intensity. Nevertheless, interference effects do not vanish when the path difference is greater than the coherence length, but they manifest themselves as a modulation of the spectrum. Classical optics is familiar with this effect for a long time [6] and recently increased attention has been paid to it [7]. However, only a few authors have dealt with this phenomenon in quantum context (e.g. [8, 9, 10]).

In this contribution we will show that both these phenomena (nonlocality or "contextuality" [11] of quantum mechanics and interference effects in spectral domain) may appear together.

2 Principle of the experiment

An outline of the proposed experimental scheme is shown in Fig. 1. Correlated photon pairs are produced by a parametric down-conversion process in a suitably cut nonlinear crystal pumped by a short-wavelength laser at frequency ω_0. Two apertures select a pair of energy correlated photons from the broadband cone behind the crystal. The signal photon beam propagates in the direction denoted by a and falls on a narrowband tunable frequency filter (e.g. Fabry-Perot) with amplitude frequency transmissivity $\mathcal{T}_f(\omega)$. The filter can be regarded as a beam-splitter with frequency dependent transmissivity $\mathcal{T}_f(\omega)$ and reflectivity $\mathcal{R}_f(\omega)$. Than the annihilation operator of the output field may be written as

$$\mathsf{d}_\mathrm{a}(\omega) = \mathcal{T}_f(\omega)\mathsf{a}(\omega) + \mathcal{R}_f(\omega)\mathsf{a}_0(\omega), \tag{1}$$

where $\mathsf{a}_0(\omega)$ is the operator of the mode at the unused port. The detector D_a is assumed to be placed just behind the filter.

In the route of the idler photon, referred to as b, a Mach-Zehnder interferometer is mounted, no matter how far. The annihilation operators of the input and output modes of the interferometer are related by the the following unitary transformation ($\mathsf{b}_0(\omega)$ represents the field at the unused input port)

$$\begin{bmatrix} \mathsf{d}_1(\omega) \\ \mathsf{d}_2(\omega) \end{bmatrix} = \begin{bmatrix} \mathcal{R} & \mathcal{T} \\ \mathcal{T} & \mathcal{R} \end{bmatrix} \begin{bmatrix} \exp(-i\omega t_l) & 0 \\ 0 & \exp(-i\omega t_s) \end{bmatrix} \begin{bmatrix} \mathcal{R} & \mathcal{T} \\ \mathcal{T} & \mathcal{R} \end{bmatrix} \begin{bmatrix} \mathsf{b}(\omega) \\ \mathsf{b}_0(\omega) \end{bmatrix}, \tag{2}$$

where $\mathcal{R}$ and $\mathcal{T}$ are the amplitude reflection and transmission coefficients, respectively, which are considered the same for both beam-splitters and which are assumed to be frequency independent and to satisfy the usual unitarity relations

$$\begin{aligned} |\mathcal{R}|^2 + |\mathcal{T}|^2 &= 1, \\ \mathcal{R}^*\mathcal{T} + \mathcal{R}\mathcal{T}^* &= 0. \end{aligned} \tag{3}$$

The quantities t_l and t_s are the transit times through the longer and shorter arms. Assuming the symmetrical beam-splitters with $\mathcal{R} = i/\sqrt{2}$ and $\mathcal{T} = 1/\sqrt{2}$, one obtains the

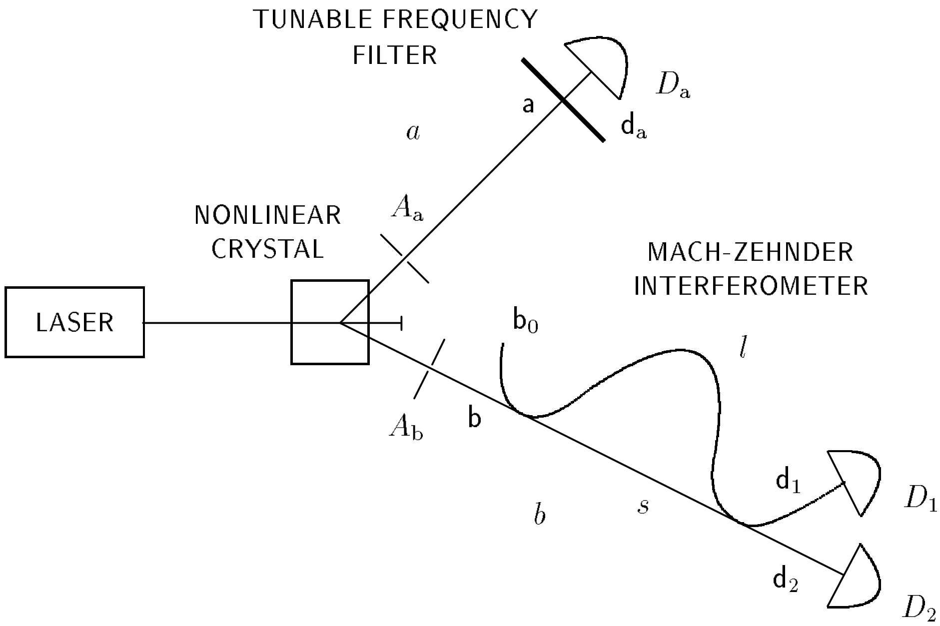

Figure 1: Proposed experimental setup. A_{a} and A_{b} are apertures selecting photon-pair beams; D_{a}, D_1, and D_2 are photodetectors; a denotes the route of the signal photon, b the route of the idler photon; l and s denote long and short arms of the interferometer.

following expression for the operators $\mathsf{d}_j(\omega)$ $[j = 1, 2]$ in terms of the input fields

$$\mathsf{d}_j(\omega) = B_j \mathsf{b}(\omega) + B_{0j} \mathsf{b}_0(\omega), \tag{4}$$

where

$$\begin{aligned} B_1(\omega) = -B_{02}(\omega) &= \tfrac{1}{2} \exp(-i\omega t_l)[\exp(i\omega\Delta t) - 1], \\ B_{01}(\omega) = B_2(\omega) &= i\tfrac{1}{2} \exp(-i\omega t_l)[\exp(i\omega\Delta t) + 1], \end{aligned} \tag{5}$$

and $\Delta t = t_l - t_s$.

Let us suppose that the level of excitation of the field is low enough so that there is a very low probability to appear more then one photon pair (at one time). We will assume for other calculations that just one pair of photons is present. The state of the field generated in the nonlinear crystal may then be written as

$$|\psi\rangle = (\delta\omega)^2 \sum_{\omega} \sum_{\omega'} \xi(\omega, \omega') \, \mathsf{a}^\dagger(\omega) \mathsf{b}^\dagger(\omega') \, |\mathrm{vac}\rangle, \tag{6}$$

where $\delta\omega$ is the mode spacing [12]; $\mathsf{a}^\dagger(\omega)$ and $\mathsf{b}^\dagger(\omega)$ are the appropriate creation operators. The function $\xi(\omega, \omega')$ characterizes the correlation between the modes a and b. This function also includes the influence of the finite width of the laser spectral line and of the finite interaction volume.

From the requirement of normalization it follows that [13]

$$(\delta\omega)^2 \sum_{\omega} \sum_{\omega'} |\xi(\omega, \omega')|^2 = 1. \tag{7}$$

Further let us consider perfect energy correlation between the signal and idler photons.

$$\xi(\omega,\omega') = \begin{cases} \eta(\omega)(\delta\omega)^{-1/2} & \text{if } \omega' = \omega_0 - \omega, \\ \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

With respect to Eq.(7), one can find that

$$\delta\omega \sum_{\omega} |\eta(\omega)|^2 = 1. \tag{9}$$

3 Calculation of detection rates

First we find the expressions for the electric fields operators $\mathsf{E}_{\mathrm{a}}(t)$ and $\mathsf{E}_j(t)$ at the detectors D_{a} and D_j $[j = 1,2]$. Designating t_a the propagation time from the crystal to the frequency filter and t_b the propagation time from the crystal to the interferometer, one may write the positive frequency parts of the mentioned operators as follows

$$\begin{aligned} \mathsf{E}_{\mathrm{a}}^{(+)}(t) &= \frac{\delta\omega}{(2\pi)^{1/2}} \sum_{\omega} \mathsf{d}_{\mathrm{a}}(\omega) \exp[-i\omega(t - t_a)] \\ &= \frac{\delta\omega}{(2\pi)^{1/2}} \sum_{\omega} [\mathcal{T}_f(\omega)\mathsf{a}(\omega) + \mathcal{R}_f(\omega)\mathsf{a}_0(\omega)] \exp[-i\omega(t - t_a)], \end{aligned} \tag{10}$$

$$\begin{aligned} \mathsf{E}_j^{(+)}(t) &= \frac{\delta\omega}{(2\pi)^{1/2}} \sum_{\omega} \mathsf{d}_j(\omega) \exp[-i\omega(t - t_b)] \\ &= \frac{\delta\omega}{(2\pi)^{1/2}} \sum_{\omega} [B_j(\omega)\mathsf{b}(\omega) + B_{0j}(\omega)\mathsf{b}_0(\omega)] \exp[-i\omega(t - t_b)]. \end{aligned} \tag{11}$$

The fields are normalized so that $\mathsf{E}^{(-)}\mathsf{E}^{(+)}$ is in units of photons per second.

Let us suppose that the detectors D_{a} and D_j have quantum efficiencies α_{a} and α_j, respectively. Then, regarding the state of the field described above, the average rates of photon counting at detectors D_1 and D_2, irrespective of whether the detector D_{a} registers a photon or not, are given by [14]

$$\begin{aligned} R_j(t) &= \alpha_j \langle\psi|\, \mathsf{E}_j^{(-)}(t)\mathsf{E}_j^{(+)}(t)\, |\psi\rangle \\ &= \alpha_j \frac{(\delta\omega)^2}{2\pi} \sum_{\omega} |\eta(\omega) B_j(\omega_0 - \omega)|^2 . \end{aligned} \tag{12}$$

For our purposes it is interesting to suppose that the transit time difference Δt arising in the interferometer is much longer than the coherence time t_{coh} of the field. In such a case no interference is expected. Using Eqs. (5) and (9), and changing the sum to an integral, one can find that

$$\begin{aligned} \delta\omega \sum_{\omega} |\eta(\omega) B_j(\omega_0 - \omega)|^2 &= \delta\omega \sum_{\omega} \frac{1}{2} |\eta(\omega)|^2 \left\{1 + (-1)^j \cos[(\omega_0 - \omega)\Delta t]\right\} \\ &= \frac{1}{2} + (-1)^j \frac{1}{2} \underbrace{\int d\omega\, \eta(\omega) \cos[(\omega_0 - \omega)\Delta t]}_{\approx 0} \approx \frac{1}{2}. \end{aligned} \tag{13}$$

The probability to detect a photon at D_1 is the same as at D_2. The last approximation in Eq.(13) may be used because the function $|\eta(\omega)|^2$ varies much more slowly than the cosine term in virtue of the assumption that $\Delta t \gg t_{\rm coh}$.

However, performing coincidence measurements with the signal from the detector $D_{\rm a}$ placed behind a narrowband filter, whose pass-band width is much less than the reciprocal transit time difference $(\Delta t)^{-1}$, one obtains something quite different.

The rate of coincidence detection or the probability density that a photon will be detected by the detector $D_{\rm a}$ just behind the tunable frequency filter in the route a at time t, and a photon will be detected by the detector D_1 or D_2 in the route b behind the interferometer at time $t+\tau$, is proportional to the correlation function of the fourth order and it is given by the formula

$$\begin{aligned} R_{{\rm a}j}(t,t+\tau) &= \alpha_{\rm a}\alpha_j \langle\psi|\, \mathsf{E}_{\rm a}^{(-)}(t)\mathsf{E}_j^{(-)}(t+\tau)\mathsf{E}_j^{(+)}(t+\tau)\mathsf{E}_{\rm a}^{(+)}(t)\,|\psi\rangle \\ &= \alpha_{\rm a}\alpha_j \frac{(\delta\omega)^3}{(2\pi)^2}\left|\sum_\omega \exp[i\omega(\tau - t_a + t_b)]\,\eta(\omega)B_j(\omega_0-\omega)\mathcal{T}_f(\omega)\right|^2 . \end{aligned} \quad (14)$$

Here again $j = 1$ or 2 and it corresponds to a count at D_1 or D_2.

As the frequency dependence of the transmissivity of the filter is assumed very narrow, we can formally put

$$\mathcal{T}_f(\omega) = \begin{cases} 1 & \text{if } \omega = \omega_f, \\ 0 & \text{otherwise,} \end{cases} \quad (15)$$

where ω_f is the central frequency of the pass band. Then, substituting into Eqs. (14), we obtain

$$R_{{\rm a}j}(t,t+\tau) = \alpha_{\rm a}\alpha_j \frac{(\delta\omega)^3}{(2\pi)^2}\left|\eta(\omega_f)B_j(\omega_0-\omega_f)\right|^2 , \quad (16)$$

Using Eq. (5), one finally obtains the following expressions for the quantity $|B_j(\omega_0 - \omega_f)|^2$ appearing in Eqs. (16)

$$\begin{aligned} |B_1(\omega_0-\omega_f)|^2 &= \tfrac{1}{2}\left\{1-\cos[(\omega_0-\omega_f)\Delta t]\right\}, \\ |B_2(\omega_0-\omega_f)|^2 &= \tfrac{1}{2}\left\{1+\cos[(\omega_0-\omega_f)\Delta t]\right\}. \end{aligned} \quad (17)$$

These formulas show that varying ω_f (i.e., tuning the filter), the values of $|B_j(\omega_0-\omega_f)|^2$ range between 0 and 1 (and always $|B_1|^2 + |B_2|^2 = 1$). That means that the coincidence detection rates are modulated in dependence on ω_f and Δt, i.e., interference appears in the remote interferometer placed in the branch b – in the route of the other photon of the pair.

4 Conclusions

As expected, no interference appears at separate measurements on Mach-Zehnder interferometer if $\Delta t \gg t_{\rm coh}$ [see Eq. (13)]. If a frequency filter prolonging the coherence length sufficiently were placed in front of the interferometer, interference would appear. Placing a (scanning) filter in front of one detector at the output of the interferometer, one could observe a frequency modulation since individual frequency components of the field (even in case of a single photon) interfere independently and the filter selects just "one" of them. However, in the case described above, there is no filter in the part containing the interferometer (i.e., in the route b); the spectral selection is done in the route a. Nevertheless, when a photon of frequency ω_f is registered at $D_{\rm a}$ the interference effects (dependent on ω_f) appear at the outputs of the interferometer in the part b (*in the sense of coincidence measurement*), as evident from Eqs. (17). It happens even if the apparatus is arranged in such a way that the spectra of both photons do not overlap!

One can consider the arrangement where the wave packet first reaches the interferometer in part b and only after this its twin comes to the filter in part a — then we can, at least in principle, choose the frequency (tune the filter) after the detection at D_j. There is no contradiction: The measurement in the route b affects the state of the field at the route a in such a way that the frequencies, which would allow the opposite result of the measurement in part b, will not be present there any longer. The measurement at b modulates the spectrum at a.

Let us emphasize that there is nothing acausal here and nothing actual propagates at superluminal velocity because the interference effects (at b) are observable only in coincidence with the event at the (distant) detector $D_{\rm a}$ and information on this event can be obtained only by conventional means. Until we compare the results from the parts a and b, we do not find any interference.

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[12] When it be technically convenient, we will assume $\delta\omega \to 0$ and employ the conversion $\delta\omega \sum_\omega \to \int d\omega$.

[13] We assume the following commutation relations between the annihilation and creation operators: $[\mathsf{a}(\omega), \mathsf{b}^\dagger(\omega')] = (\delta\omega)^{-1}$ if both operators correspond to the same mode and $\omega = \omega'$; otherwise the commutator is zero.

[14] As we can see, the counting rate has the form corresponding to statistical ensemble of individual monochromatic photons with probability distribution $|\eta(\omega)|^2$. Indeed, the density matrix describing the subsystem (b), which one obtains tracing of $|\psi\rangle\langle\psi|$ over the subsystem (a), has a diagonal form: $\hat{\rho}_b = (\delta\omega)^2 \sum_\omega |\eta(\omega)|^2\, \mathsf{b}^\dagger(\omega_0 - \omega)\, |\mathrm{vac}\rangle\langle\mathrm{vac}|\, \mathsf{b}(\omega_0 - \omega)$. Such a form of the density matrix of the subsystem is a consequence of the perfect correlation between the idler and signal photons.