Effective atom-light beamsplitter and atom-optical cluster states

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Outline

- Effective atom-light beamsplitter intteraction
- Mesoscopic atomic entanglement concentration
- Continuous variable atom-light cluster states
- Conclusions

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- Elements for quantum repeater
  (entanglement concentration, etc)

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- Quantum interface between light and matter
- Elements for quantum repeater
  (entanglement concentration, etc)
- Elements for small scale quantum processor
  (cluster states, non-Gaussian states, etc)

Defining the variables: Atoms

Colective spin of the atomic ensemble: $\vec{J} = \sum_{m=1}^{N_a} \vec{j}^{(m)}$
(with N_a atoms)

Quantum: $[\hat{J}_y, \hat{J}_z] = i\hat{J}_x$.

Strongly polarized atoms, spin aligned along x: replace $\hat{J}_x$ by $\langle J_x \rangle$.
Define atomic canonical "position" and "momentum":

$$\hat{q}_A = \frac{\hat{J}_y}{\sqrt{\langle J_x \rangle}}, \qquad p_A = \frac{\hat{J}_z}{\sqrt{\langle J_x \rangle}}$$

Defining the variables: Light

Quantum Stokes operators describe polarization:

$$\hat{S}_x = \frac{1}{2}\left(\hat{a}_x^\dagger \hat{a}_x - \hat{a}_y^\dagger \hat{a}_y\right) \qquad \hat{S}_y = \frac{1}{2}\left(\hat{a}_x^\dagger \hat{a}_y + \hat{a}_y^\dagger \hat{a}_x\right)$$

$$\hat{S}_z = \frac{1}{2i}\left(\hat{a}_x^\dagger \hat{a}_y - \hat{a}_y^\dagger \hat{a}_x\right) \qquad [\hat{S}_y, \hat{S}_z] = i\hat{S}_x$$

Strongly polarized light, along x: replace $\hat{S}_x$ by $\langle S_x \rangle$.
Define light canonical "position" and "momentum":

$$\hat{q}_L = \frac{\hat{S}_y}{\sqrt{\langle S_x \rangle}}, \qquad \hat{p}_L = \frac{\hat{S}_z}{\sqrt{\langle S_x \rangle}}$$

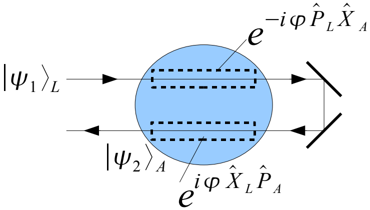
Defining the interaction

Quantum Non-Demolition (QND) interaction between light beam and mesoscopic atomic ensemble: mapping of quantum state of light onto atoms (and vice versa)

$$H \propto S_z J_z \qquad \hat{H} = \kappa \hat{X}_L \hat{P}_A$$

B. Julsgaard, A Kozhekin, E. S. Polzik, Nature, Vol **413**, *400-403 (2001); Quantum interface between light and atomic ensembles, K. Hammerer, A.S. Sorensen, E.S. Polzik, review to appear in Rev. Mod. Phys., arXiv:0807.3358*

Effective light-atom beamsplitter



First interact via the QND Hamiltonian $\hat{H}_1 = \kappa \hat{P}_L \hat{X}_A$. Then the quadratures are rotated and the light field is reflected back into the atomic ensemble: $\hat{H}_2 = -\kappa \hat{X}_L \hat{P}_A$.

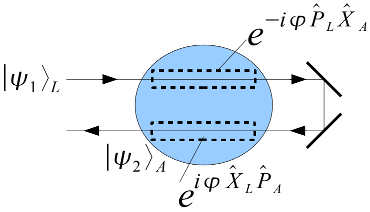
The interaction of this system:

$$\hat{U} = e^{i\varphi \hat{X}_L \hat{P}_A} e^{-i\varphi \hat{P}_L \hat{X}_A}$$

True beamsplitter:

$$\hat{U} = e^{i\varphi \hat{X}_L \hat{P}_A - i\varphi \hat{P}_L \hat{X}_A}$$

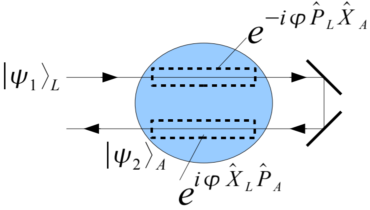
(φ = the coupling constant κ integrated over the interaction time of each pass-through)



The quadratures of the atoms have been written onto the light and vice versa:

$$|q_1\rangle_L\,|q_2\rangle_A \longrightarrow |q_1 + \phi q_2\rangle_L|(1-\phi^2)q_2 - \phi q_1\rangle_A$$
$$|p_1\rangle_L\,|p_2\rangle_A \longrightarrow |(1-\phi^2)p_1 + \phi p_2\rangle_L|p_2 - \phi p_1\rangle_A.$$

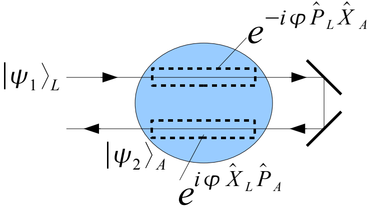
- How good this approximates a beamplitter interaction?



The quadratures of the atoms have been written onto the light and vice versa:

$$|q_1\rangle_L |q_2\rangle_A \rightarrow |q_1 + \phi q_2\rangle_L |(1-\phi^2)q_2 - \phi q_1\rangle_A$$
$$|p_1\rangle_L |p_2\rangle_A \rightarrow |(1-\phi^2)p_1 + \phi p_2\rangle_L |p_2 - \phi p_1\rangle_A.$$

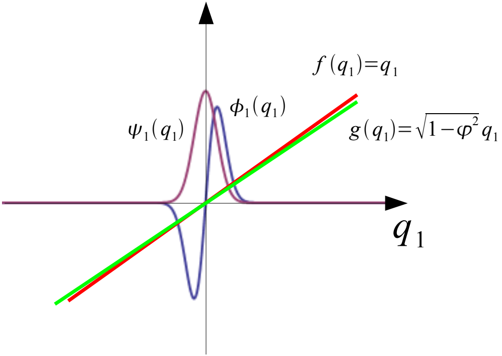
- Obvious: for $\phi << 1$, weak coupling: $1 \approx 1 - \phi^2 \approx \sqrt{1-\phi^2}$.



The quadratures of the atoms have been written onto the light and vice versa:

$$|q_1\rangle_L\,|q_2\rangle_A \to |q_1+\phi q_2\rangle_L|(1-\phi^2)q_2-\phi q_1\rangle_A$$
$$|p_1\rangle_L\,|p_2\rangle_A \to |(1-\phi^2)p_1+\phi p_2\rangle_L|p_2-\phi p_1\rangle_A.$$

- Now note: need another set of constraints on the input states: $(1-\phi^2)q_2 \approx \sqrt{1-\phi^2}q_2$ and $q_1 \approx \sqrt{1-\phi^2}q_1$ are good approximations for $\phi << 1$ while q_1 and q_2 are small. But: q_1 and q_2 assume values over the entire real line. Hence bad beamsplitter fidelity unless we restrict the type of input states.

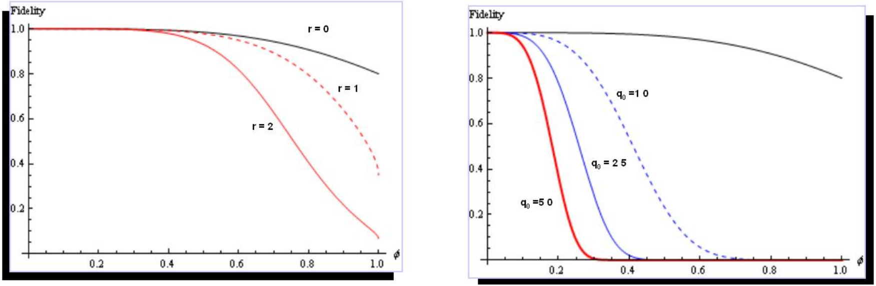


Eligible wave-functions

e.g. $\psi_1(q_1)$, $\phi_1(q_1)$

must be negligible in the regions of q_1 where the previous approximations are not satisfied (same for q_2).

This joint constraint on the input states and the coupling between systems is reminiscent of the approximations in the weak measurement formalism of Aharonov *et al*.



Fidelity: overlap of the Wigner functions for the ideal beamsplitter and for the QND protocol:

$$\mathcal{F} = (2\pi)^2 \int_{\infty}^{\infty} \int_{\infty}^{\infty} dq_1 dp_1 dq_2 dp_2 \mathcal{W}_{AL} \mathcal{W}_{BS}$$

Left: the fidelity for a squeezed vacuum input, r - squeezing parameter.

Right: Coherent state input with the coherent amplitude q_0.

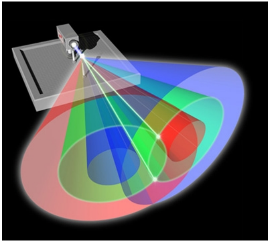
Applications of the atom-light BS

- $\phi << \mathbf{1}$:
  entanglement concentration for two atomic ensembles using photom subtraction scheme.

- $\phi \rightarrow \mathbf{0.5}$ (if the fidelity of the interaction remains high!):
  This interaction approaches a 50 : 50 beamsplitter. This would be very interesting as it could be possible to see purely quantum mechanical effects on the atoms. Potentially this could be used to create an atomic cat state.

(Macroscopic atomic entanglement: see K. Hammerer, A.S. Sorensen, E.S. Polzik, review to appear in Rev. Mod. Phys., arXiv:0807.3358)

Entanglement concentration of the TMSV



Two-mode squeezed vacuum (TMSV)

$$|\zeta(\lambda)\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}(-\lambda)^n|n\rangle \otimes |n\rangle$$

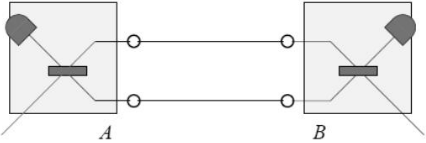


FIG. 1: A single step of the protocol. Two pairs of entangled two-mode states are mixed locally at 50:50 beam splitters and absence or presence of photons is detected in one of the output arms on both sides.

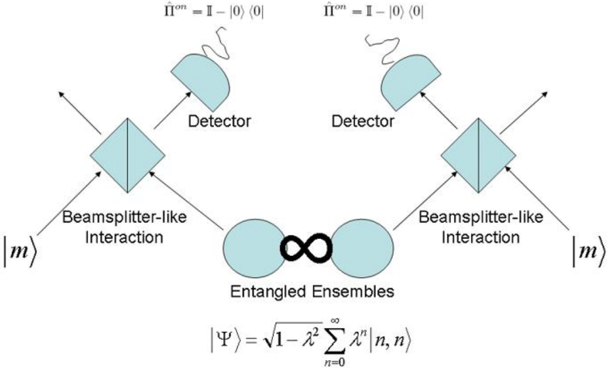
Entanglement concentration by photon subtraction

J. Eisert, S. Scheel, and M.B. Plenio, Phys. Rev. Lett. 89, 137903 (2002)

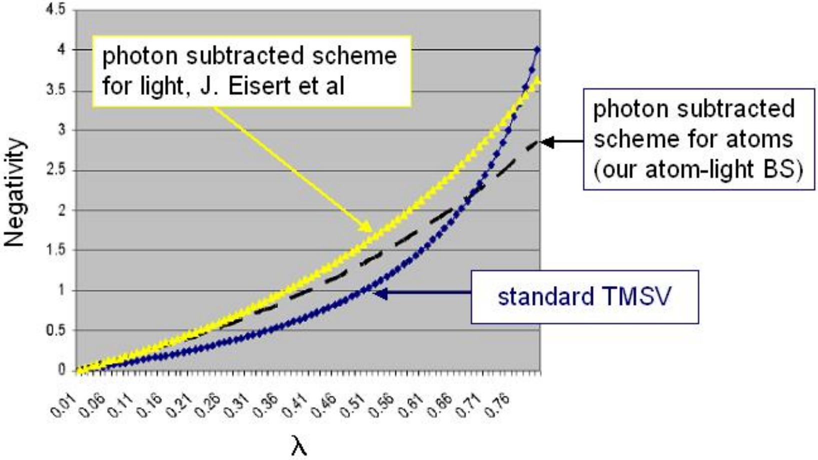
Entanglement concentration of atomic "TMSV"

Fock states $|n\rangle$ for the atomic ensembles are superpositions of all possible scenarios in which eexactly n atoms have spin $m_J = +1/2$. If each ensemble has N_a atoms:

$$|\zeta(\lambda)\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{2N_a}(-\lambda)^n|n\rangle_1 \otimes |n\rangle_2$$



Increase in entanglement content: Negativity



Entanglement of the two ensembles is increased above that of the usual TMSV up to $\lambda \sim 0.69$. The value of the interaction strength is $\varphi = \sqrt{0.1}$.

Hybrid CV atom-light cluster states

Cluster states:

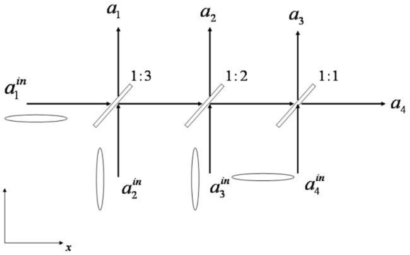
H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, *910 (2001)*

First idea for CV:

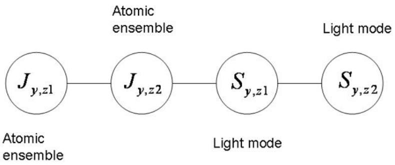
J. Zhang, S. L. Braunstein, Phys. Rev. A. 73, 032318 (2006).

Further developments: *P. van Loock and co-wokers*

CV cluster state for light:



CV atom-light cluster:



Hybrid CV atom-light cluster states

One of the possible definitions:

$$(\hat{p}_a - \sum_{b \in N_a} \hat{x}_b) \rightarrow 0$$

Re-written for our QND interactions:

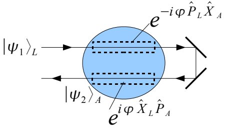
$$V(\hat{J}_{y1} - \hat{J}_{y2}) \rightarrow 0 \qquad V(\hat{J}_{y2} - \hat{J}_{z1} - \hat{S}_{z1}) \rightarrow 0$$

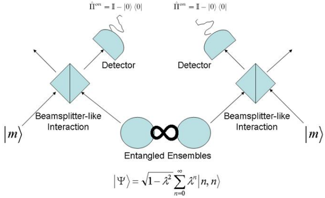
$$V(\hat{S}_{y1} - \hat{J}_{z2} - \hat{S}_{z2}) \rightarrow 0 \qquad V(\hat{S}_{y2} - \hat{S}_{z1}) \rightarrow 0$$

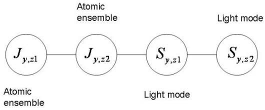
Using QND interactions and entangling pulses one can create a 4-node cluster of 2 atomic ensembles and 2 light modes

Conclusions

- Effective beamsplitter interaction between light and atomic ensemble via QND Hamiltonian: atom-light beamsplitter (BS)
- Entanglement concentration of macroscopic atomic ensembles using photon subtraction scheme based on atom-light BS.
- Hybrid CV atom-light custer states exploiting QND interaction betweeen light and atoms







Quantum Optics & Quantum Information

St. Andrews Univ

- Theoretical Quantum Optics: U. Leonhardt, T. Philbin, (T. Tyc), S. Robertson
- Theoretical Quantum Information: N. Korolkova, (T. Tyc), (L. Mišta), R. Tatham, D. Milne
- Experimental Quantum Optics: F. Koenig, C. Kuklewicz, S. Hill
- Quantum Degenerate Gases: D. Cassettari, G. Smirne, G. Bruce, L. Torralbo Campo

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