Presentation of Activities in Continuous-Variable QIPC

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Theme 1

What is so special about Gaussian states ?

- A quantum de Finetti theorem in phase space

A. Leverrier and N. J. Cerf, Phys. Rev. A 80 (2009) 010102(R)

- Investigating Hudson's theorem for mixed states

A. Mandilara, E. Karpov, and N. J. Cerf, Phys. Rev. A 79 (2009) 062302.

Quantum de Finetti theorem (for qubits)

Invariant states under permutations

1   2   3   · · · · · ·   ...   ...   N

$$\Pi\,\rho^{(n)}\,\Pi = \rho^{(n)}$$

permutation

asymptotically close to

traced over

$1 \otimes 2 \otimes 3 \otimes \cdots \otimes n$

(mixture of) i.i.d. states

$\sigma^{\otimes n}$

Special role of i.i.d. states, e.g. in quantum cryptography

Quantum de Finetti theorem (for CV)

Invariant states under rotations in phase space

1 2 3 · · · · · · · ... ... N

$$U\,\rho^{(n)}\,U^{*}=\rho^{(n)}$$

Gaussian symplectic operation

asymptotically close to

traced over

$1 \otimes 2 \otimes 3 \otimes \cdot\cdot\cdot\cdot\cdot\cdot\cdot \otimes n$

(mixture of)
Gaussian thermal states

$$\mu_G^{\otimes n}$$

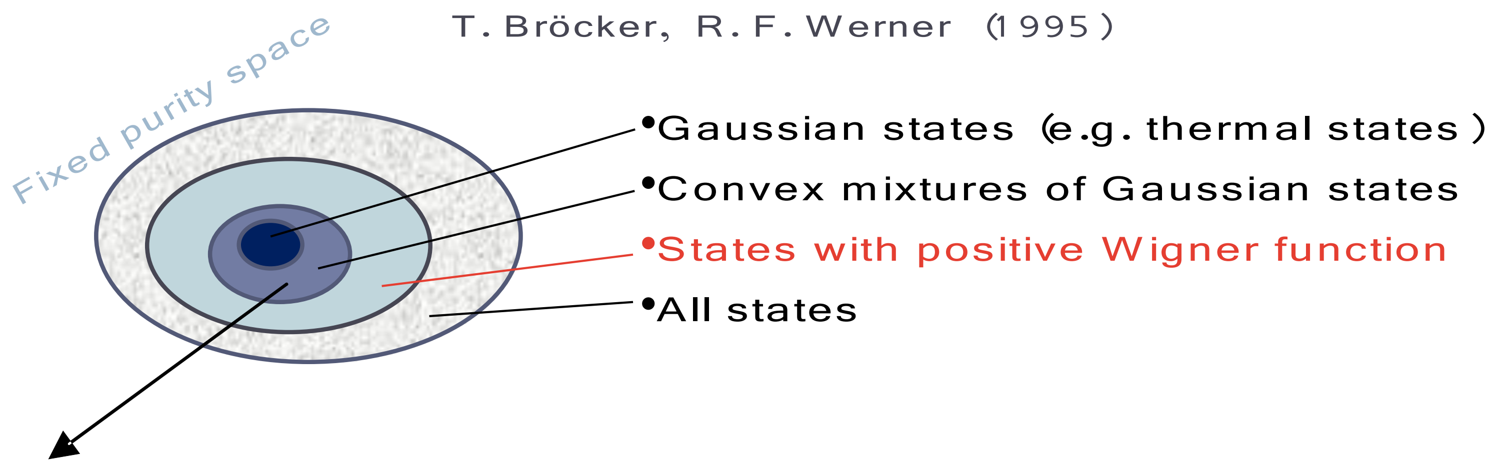
Special role of <u>Gaussian states</u>, e.g. in quantum cryptography

Hudson theorem : special role of Gaussian (pure) states

A pure state has a non-negative Wigner function <u>iff</u> it is a Gaussian state

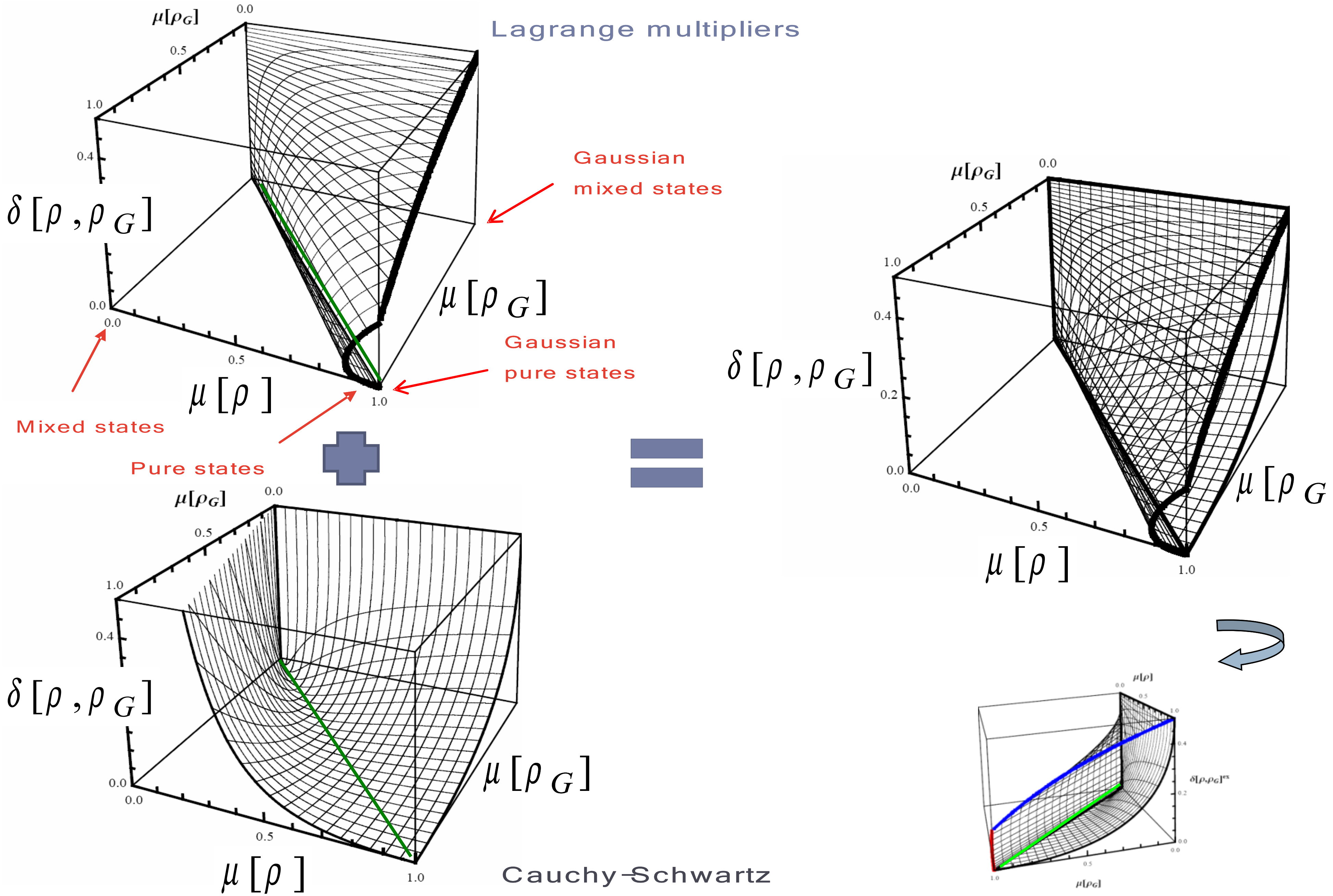
What about <u>mixed states with W(x,p)>0</u> ???

T. Bröcker, R. F. Werner (1995)



e.g., non-Gaussian mixture of Gaussian states:

$$W(x,p) = \sum_i w_i \underbrace{W_G^i(x,p)}_{\geq 0} \geq 0 \quad \text{with} \quad \sum_i w_i = 1 \qquad w_i \geq 0$$


Lagrange multipliers
Gaussian
mixed states
Gaussian
pure states
Mixed states
Pure states
Cauchy-Schwartz
$\mu[\rho_G]$
$\delta[\rho,\rho_G]$
$\mu[\rho]$
$\mu[\rho_G]$
$\delta[\rho,\rho_G]$
$\mu[\rho]$
$\mu[\rho_G]$
$\delta[\rho,\rho_G]$
$\mu[\rho]$
$\mu[\rho]$
$\delta[\rho,\rho_G]^{ex}$
$\mu[\rho_G]$

Collaboration – theme 1

Katerina Mandilara, Evgueni Karpov, NJC (ULB)
&
Anthony Leverrier, Philippe Grangier (CNRS/IO)

Theme 2

Quantum bit commitment with continuous variables ?

- A no-go theorem on Gaussian QBC

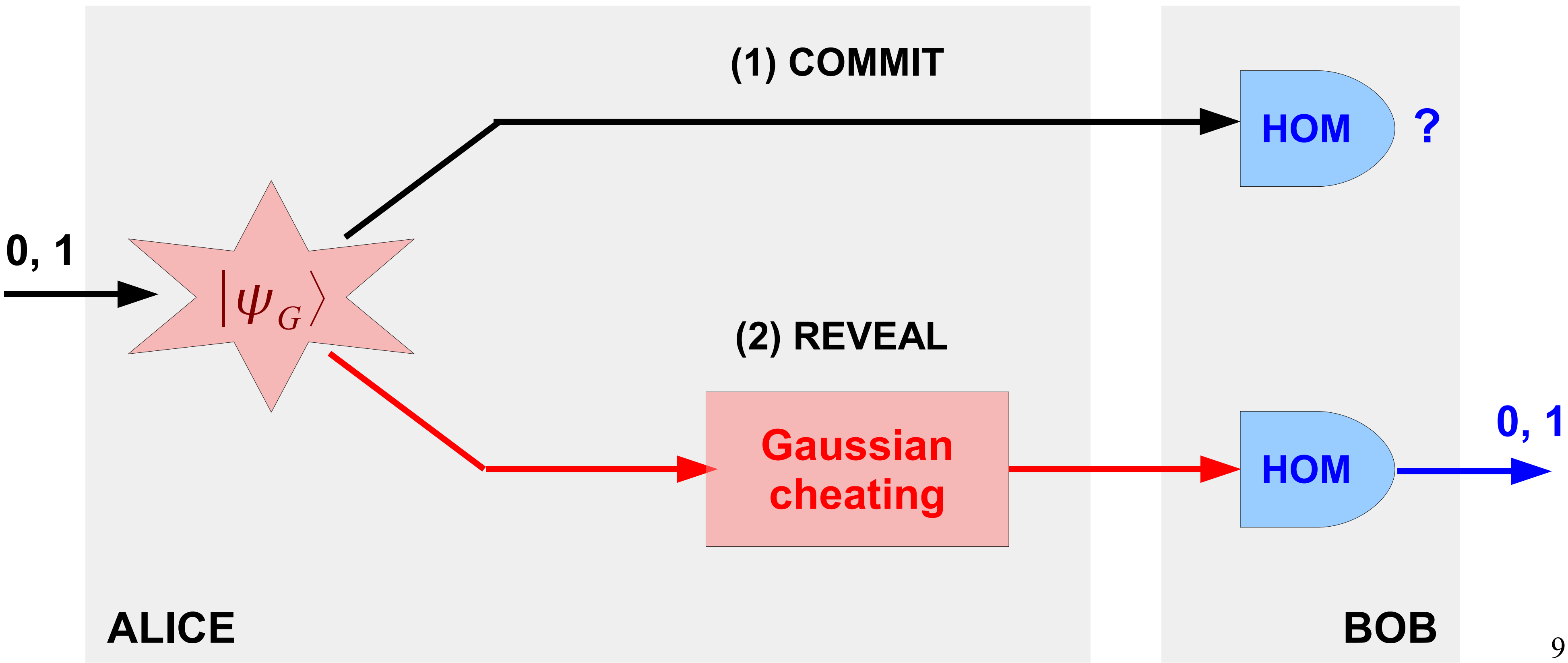
  L. Magnin, F. Magniez, A. Leverrier, and N. J. Cerf, arXiv:0905.3419 [quant-ph]

- A potentially realizable non-Gaussian protocol

  (based on photon subtraction)

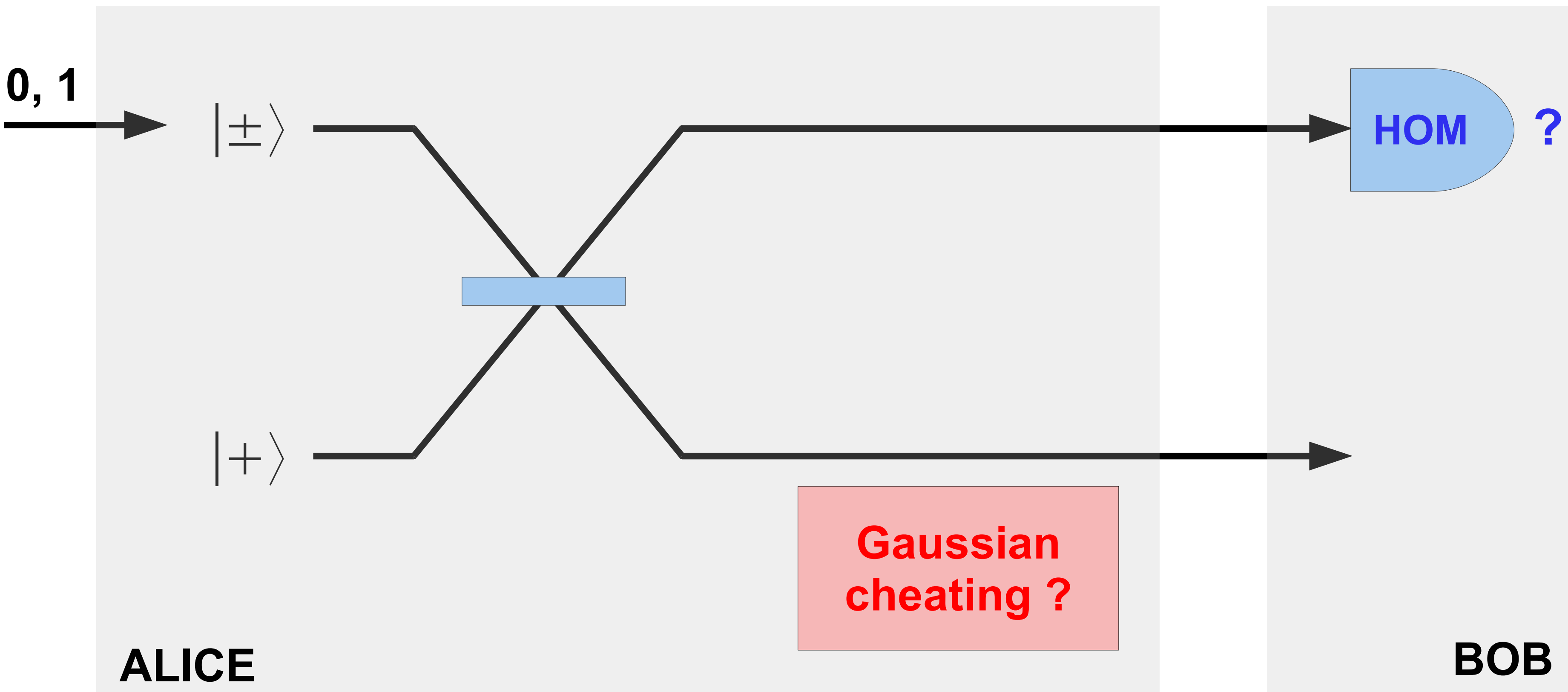
Strong no-go theorem for Gaussian QBC

QBC is well known to be impossible unless one includes restrictions, such as bounded memory; here, restriction to Gaussian operations



Non-Gaussian QBC protocol

Use of cat states: $|\pm\rangle \propto |\alpha\rangle \pm |-\alpha\rangle$



$(P_{Alice\,cheats}, P_{Bob\,cheats})$ is outside the allowed region for qubit-QBC

Collaboration – theme 2

Loïck Magnin, Xavier Lacour, NJC (ULB)
&
Anthony Leverrier, Philippe Grangier (CNRS/IO)
&
Frédéric Magniez (LRI, Orsay, France)

Theme 3

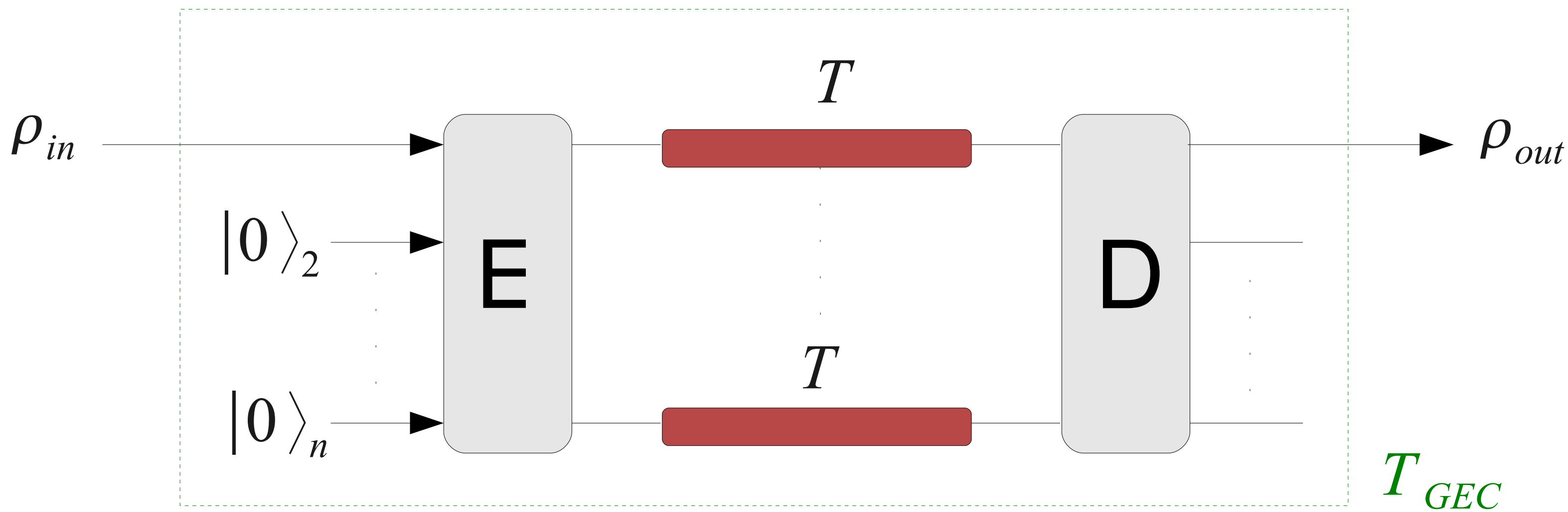
Preserving quantum optical coherence against line losses ?

- No-go for Gaussian quantum error correction

J. Niset, J. Fiurasek, and N. J. Cerf, Phys. Rev. Lett. 102 (2009) 120501.

- A quantum “erasure” correcting code
  (against non-Gaussian fluctuating loss)

Gaussian quantum error correction (GEC)



T with matrices M, N ➡ T_{GEC} with matrices M_{GEC}, N_{GEC}

Impossible to achieve GEC $[T \rightarrow T_{GEC}]$ such that $E_D[T_{GEC}] < E_D[T]$

entanglement degradation $E_D[T] = min\left(\frac{det\,N}{(1+det\,M)^2}, 1\right)$ $\quad 0 \leq E_D \leq 1$

Quantum error / erasure correcting code (QECC)

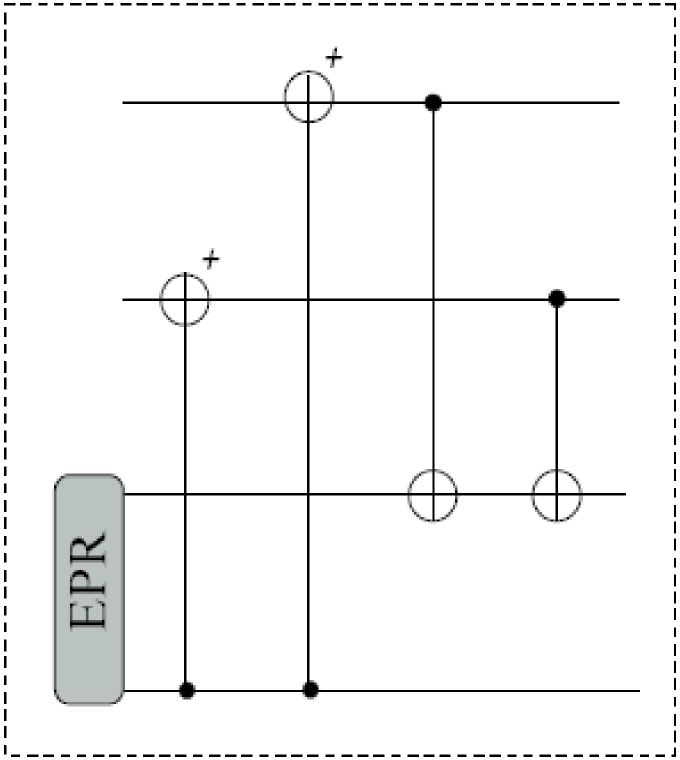
"Erasure" ≡ error of which location is known

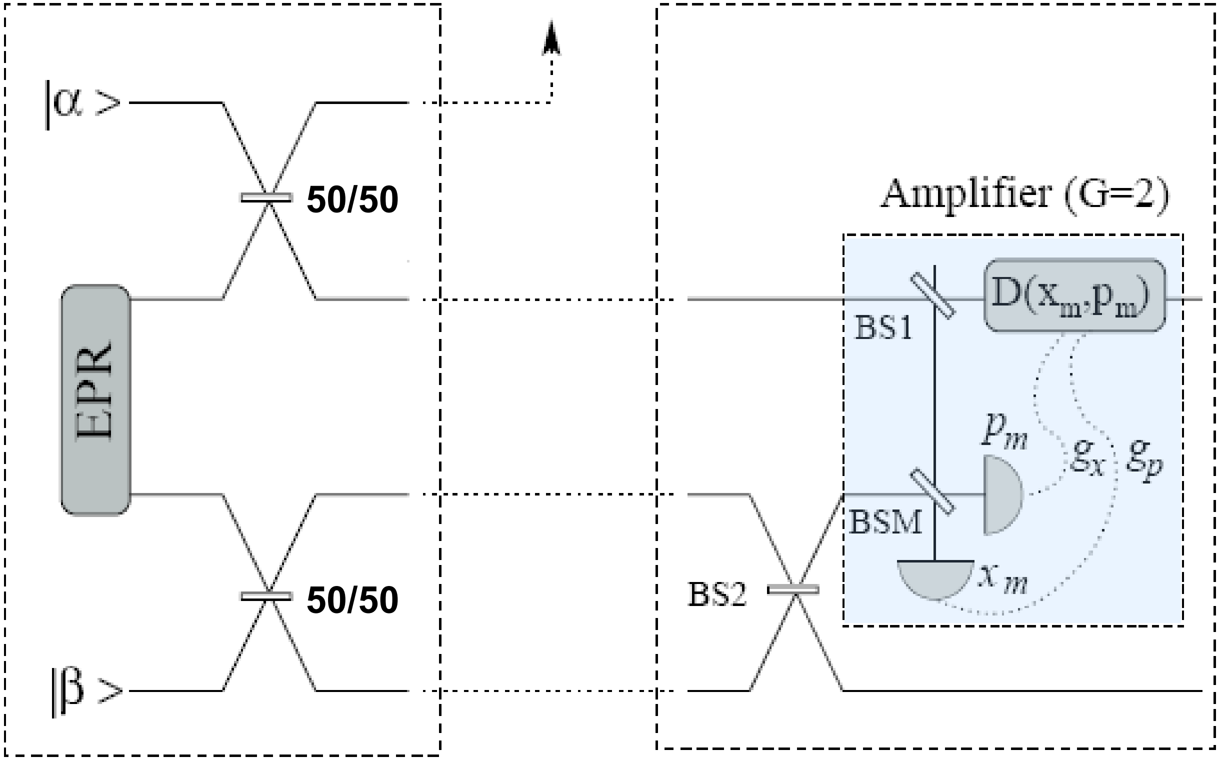
2-to-4 qubits QECC

Grassl *et al.*, PRA **56**, 33 (1997).

2-to-4 modes continuous-variable QECC

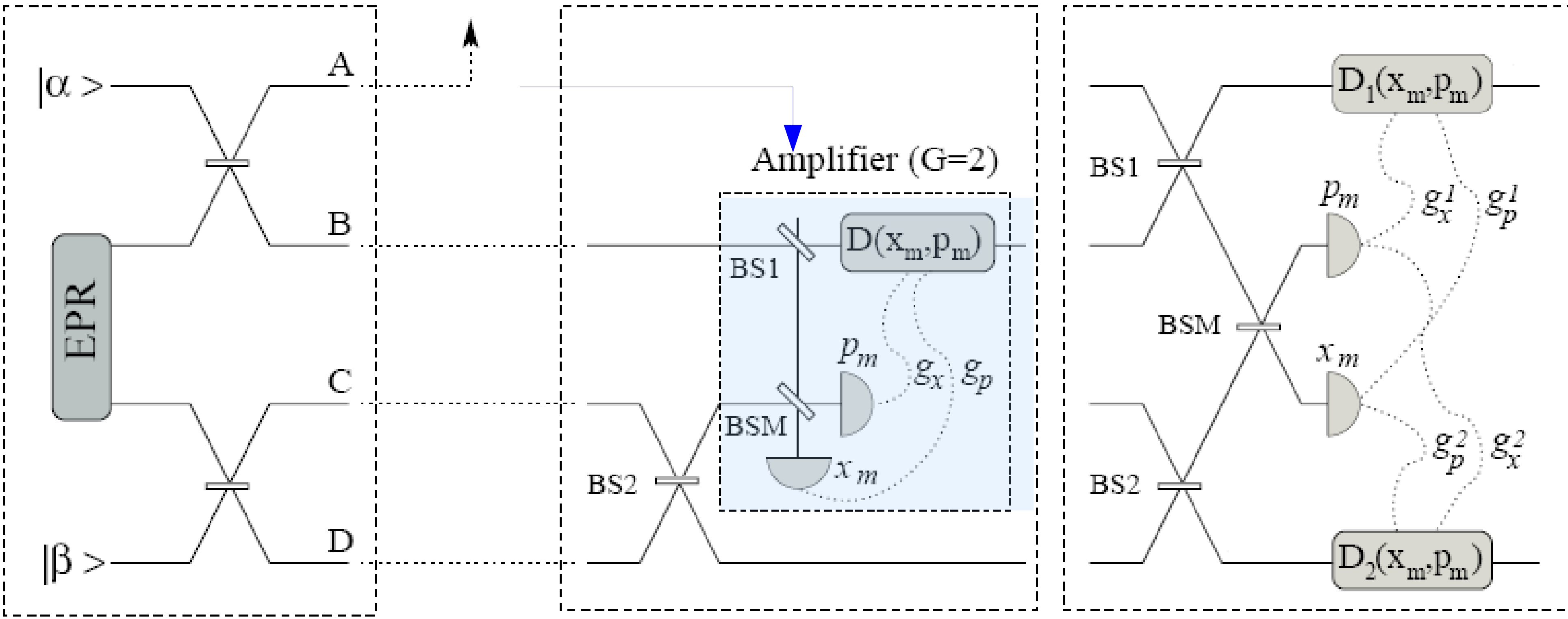
Niset, Andersen, and Cerf, PRL **101**, 130503 (2008).





Continuous-variable C-NOT gate: $|x\rangle|y\rangle \rightarrow |x\rangle|x+y\rangle$ (= QND coupling)

Decoder works for all possible erasure locations



	(g_1^x, g_1^p)	(g_2^x, g_2^p)
loss of A	$(-\sqrt{2}, -\sqrt{2})$	$(0,0)$
loss of B	$(\sqrt{2}, \sqrt{2})$	$(0,0)$
loss of C	$(0,0)$	$(\sqrt{2}, -\sqrt{2})$
loss of D	$(0,0)$	$(-\sqrt{2}, \sqrt{2})$

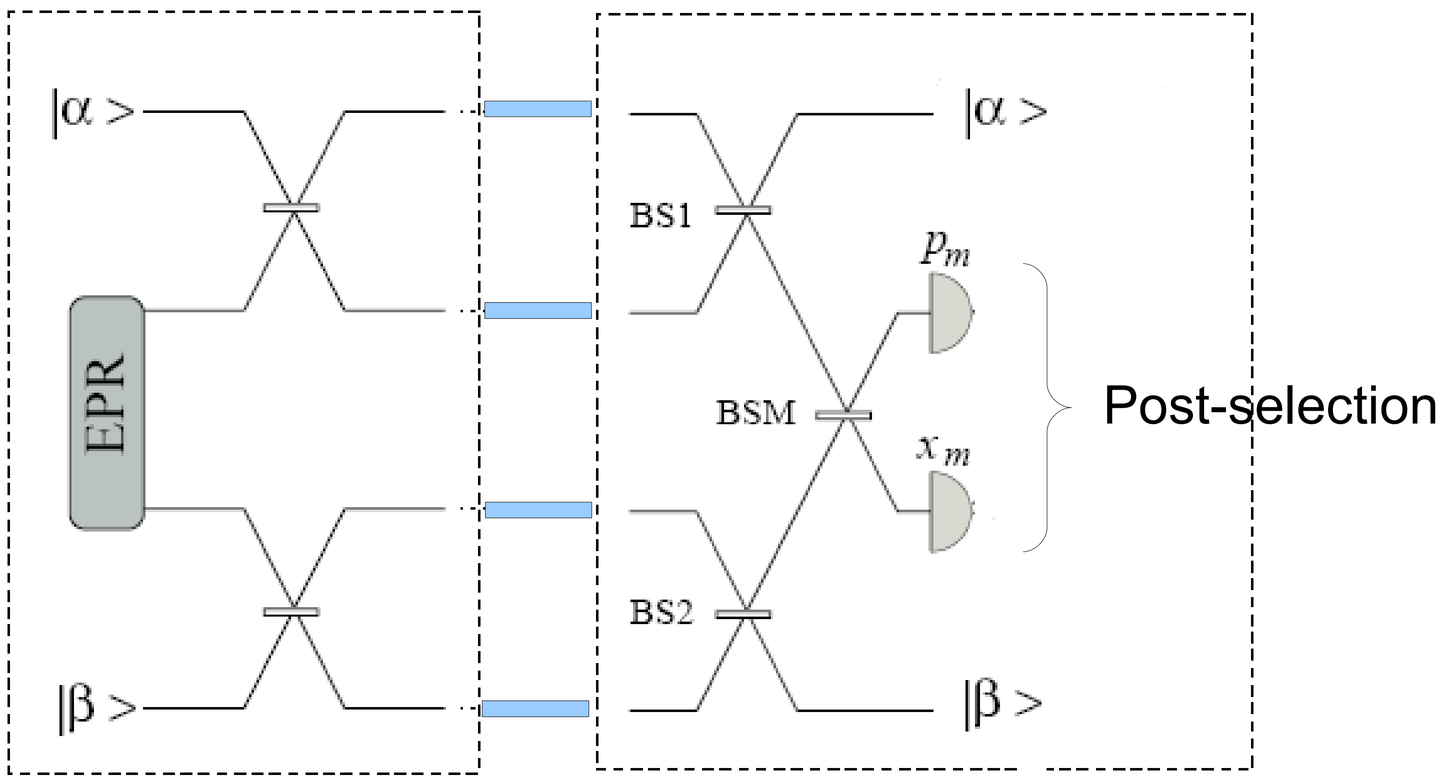
Erasure may be probed via a twin mode (polarization, frequency,...)

Probabilistic scheme

Each mode transforms as

$$\rho \rightarrow (1-p_e)\,\rho + p_e |0\rangle\langle 0|$$

with p_e = erasure probability

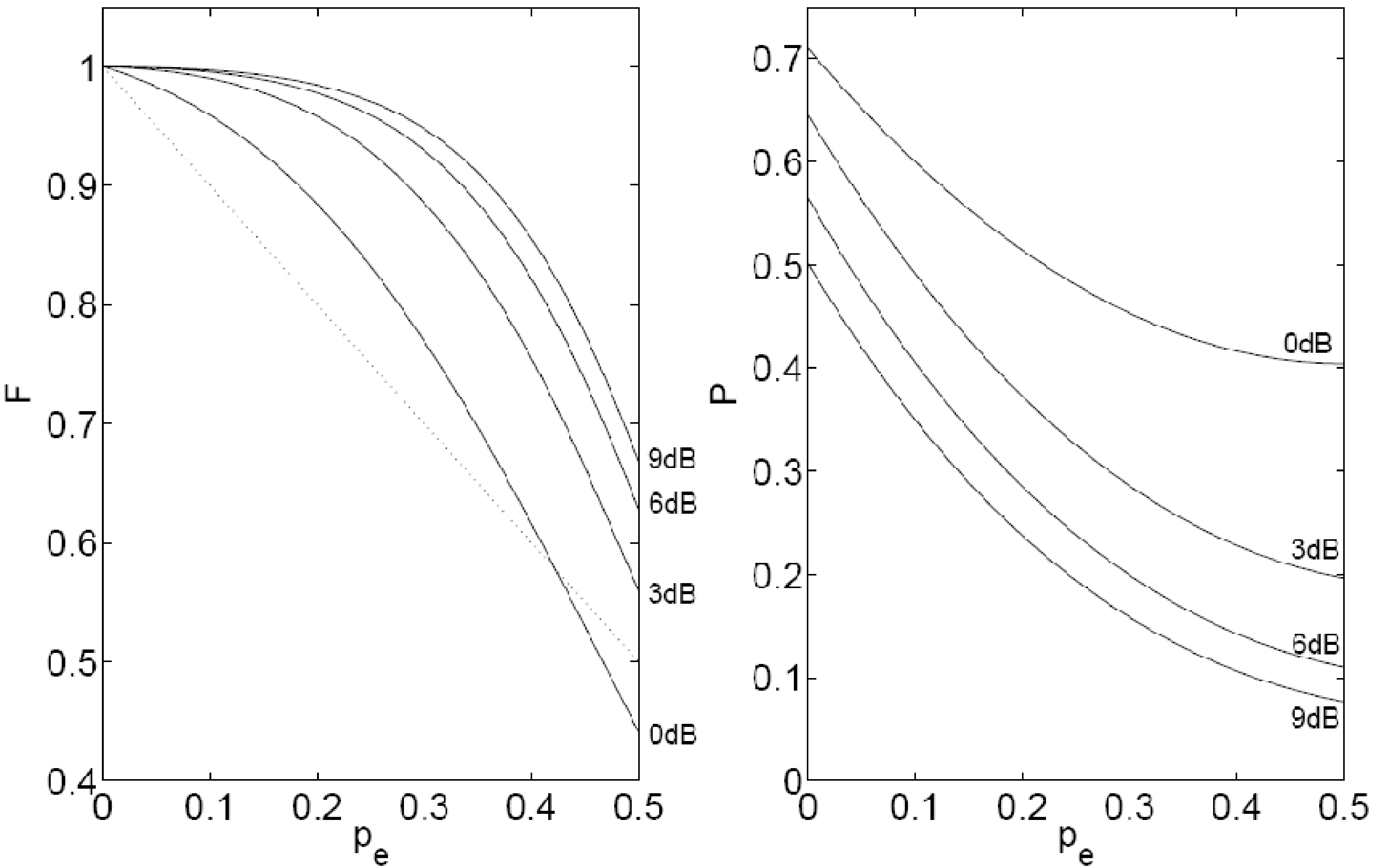


Post-selection: keeping events with $x_m \leq x_{th}$ and $p_m \leq p_{th}$

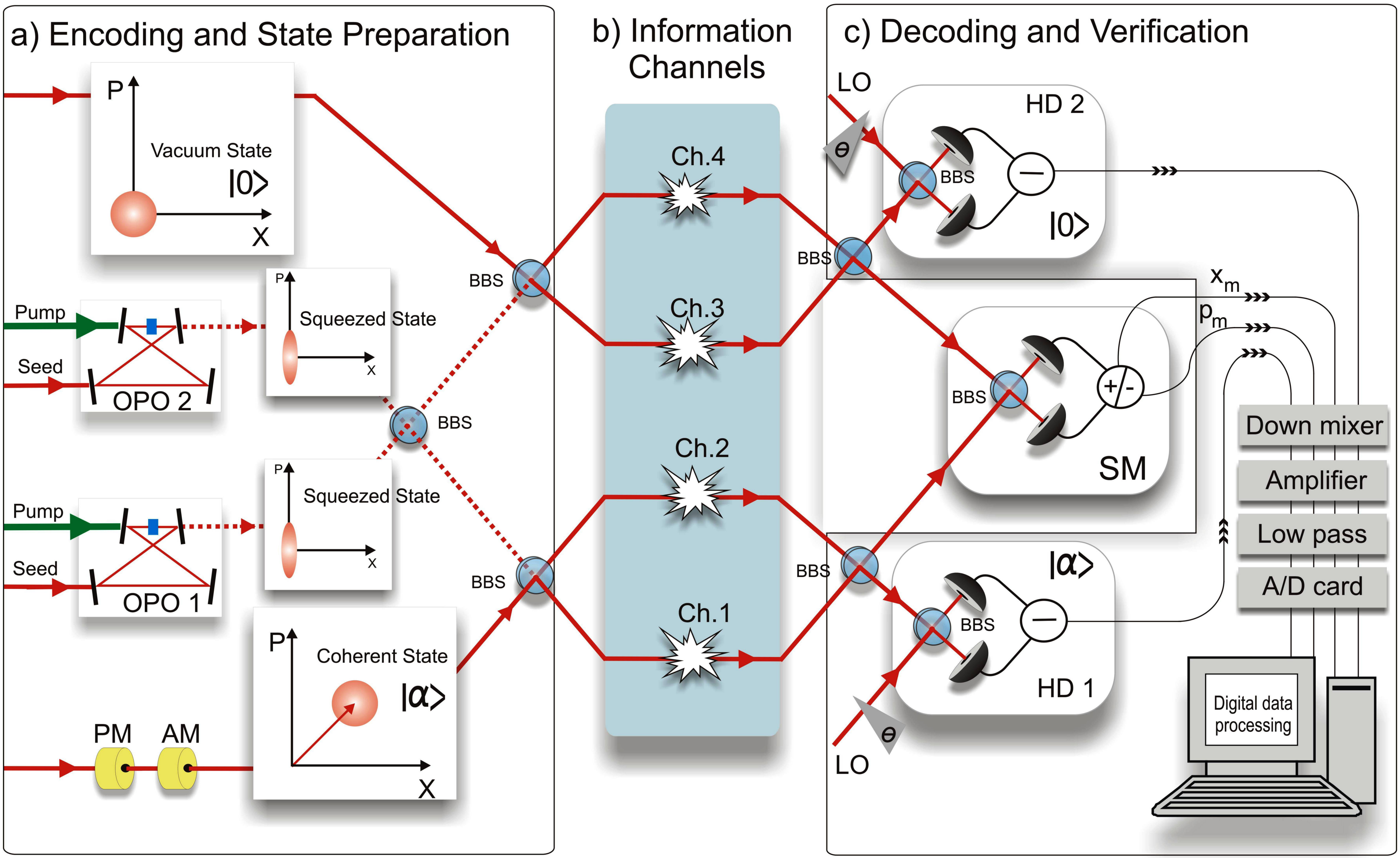
Erasure filtering

Single-mode fidelity

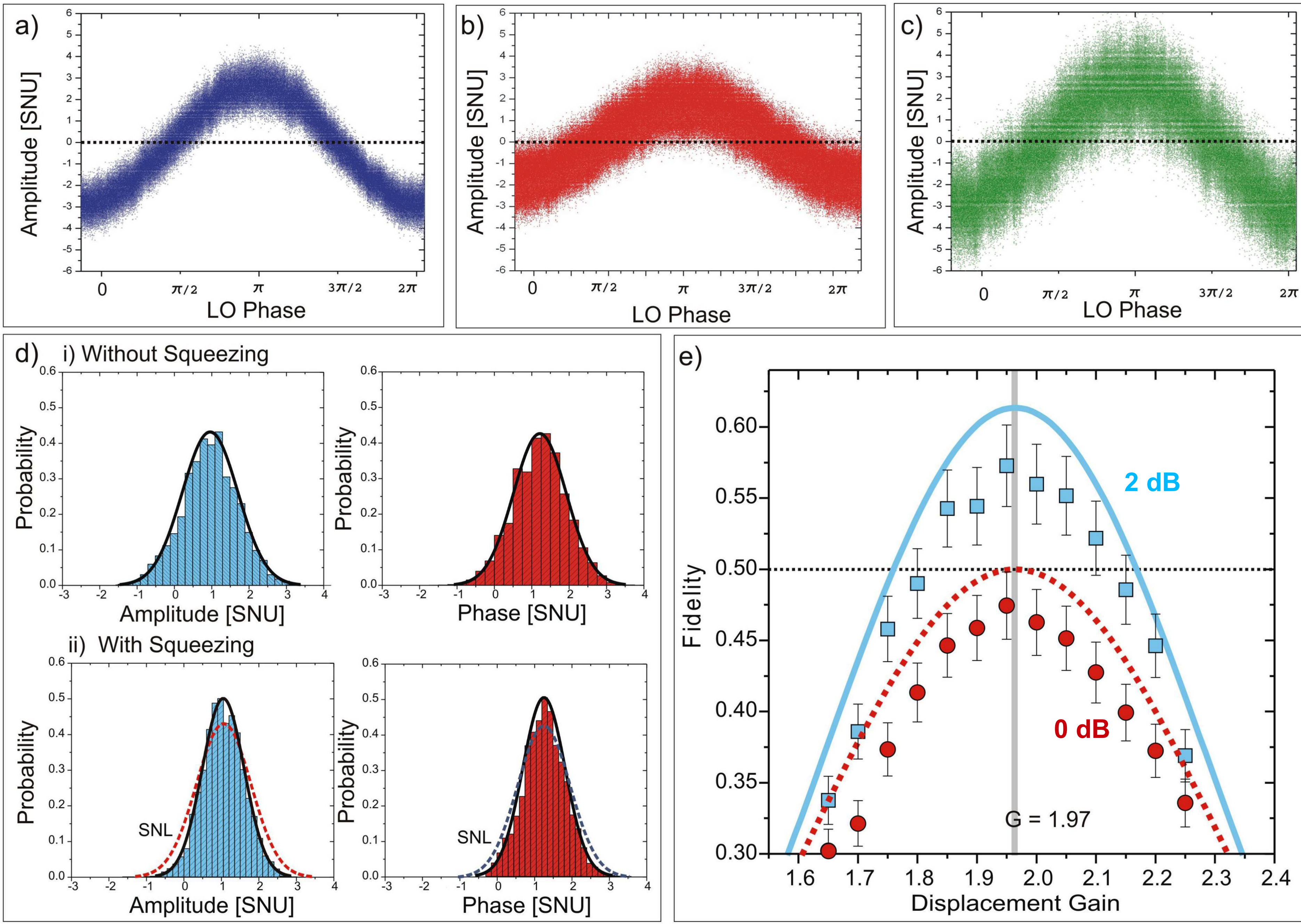
Probability of success



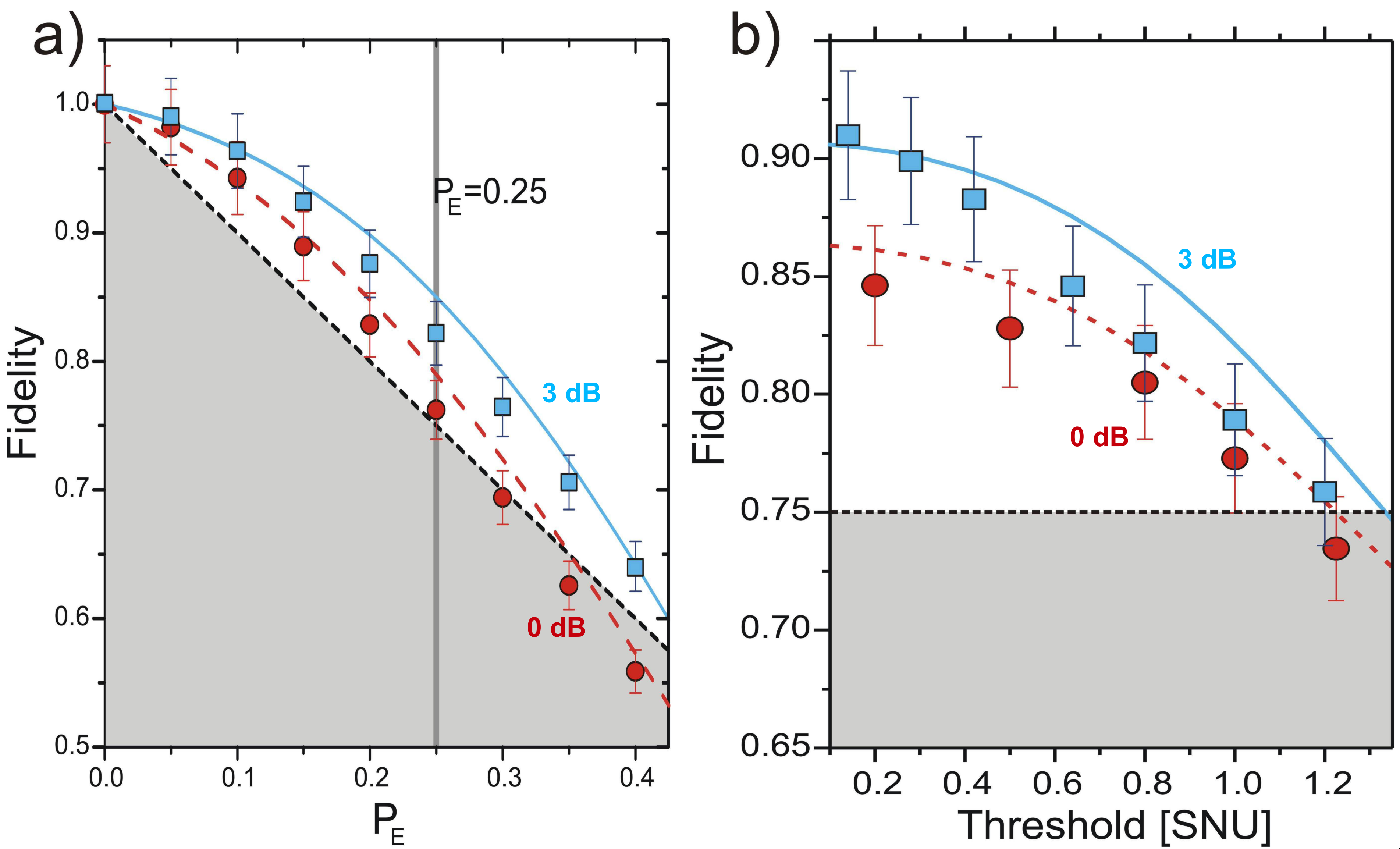
Experimental optical setup (in CW)



Experimental results for (deterministic) QECC



Experimental performances of erasure filtering



Collaboration – theme 3

Joint work with:

Julien Niset, Joachim Schäfer, NJC (ULB)
&
Jaromir Fiurasek (UP)
&
Ulrik Andersen, Metin Sabuncu, Alexander Huck (DTU)
&
Gerd Leuchs, Mikael Lassen (FAU)

Possible future themes ?

- CV quantum coin tossing
- CV fault tolerant quantum computation
  (concatenation of CV-QECC)
- CV quantum algorithm (Deutsch-Josza, Grover)
- Bipartite extension of CV quantum de Finetti
- CV quantum non locality