

Quantum Interface for Nanomechanics and Atomic Ensembles

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We suggest to interface nanomechanical systems via an optical quantum bus to atomic ensembles, for which means of high precision state preparation, manipulation and measurement are available. This allows for a Quantum Non-Demolition Bell measurement, projecting the coupled system atomic ensemble - nanomechanical resonator into an entangled state. The entanglement is observable even for nanoresonators initially well above their ground states and can be utilized for teleportation of states from an atomic ensemble to the mechanical system. Because of the rich toolbox readily available for both of these systems, we expect the interface to give rise to a variety of new quantum protocols.

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Opto- and electro-nanomechanical systems [1], representing cold high-Q oscillators coupled to optical cavities or electrical circuits, are rapidly approaching the regime where quantum aspects are important [2, 3, 4, 5]. It remains one of the key challenges of nanomechanics to develop both the tools for a preparation and manipulation of quantum states as superposition and entangled states, and implement quantum measurements. Motivated by the remarkable achievements with atomic ensembles [6, 7, 8, 9], which allow for high-fidelity preparation and readout, and laser manipulation of atomic states as long-lived quantum memory, we propose below a quantum interface between atomic ensembles and opto-mechanical systems, where light plays the role of a quantum bus.

The coupling of an atomic ensembles to light can in many cases be described in terms of two bosonic modes. Field operators $[a_c, a_c^\dagger] = 1$ correspond to a cavity mode or a propagating pulse. For an ensemble of two level atoms the mode operators $[a_a, a_a^\dagger] = 1$ describe collective excitations on top of a fully polarized initial state. In this system it is possible to engineer a number of important interactions, e.g. the beam splitter Hamiltonian $H_{BS} \propto a_a^\dagger a_c + a_a a_c^\dagger$, used for coherent storage and retrieval of light [6], and the down-conversion Hamiltonian $H_{DC} \propto a_a a_c + a_a^\dagger a_c^\dagger$, used in combination with single photon detections for probabilistic generation of light-ensemble and ensemble-ensemble entanglement [7]. Of particular relevance for the following is

$$H_{ac} = \Omega a_a^\dagger a_a + \lambda (a_a - a_a^\dagger)(a_c - a_c^\dagger), \quad (1)$$

where the first term takes into account a possible energy splitting Ω of the two atomic levels, which can be positive or negative. For degenerate levels, H_{ac} reduces to the quantum non-demolition (QND) interaction $H_{QND} = H_{BS} - H_{DC}$, used for QND measurements of atomic spins [8]. The full Hamiltonian H_{ac} , in combina-

tion with homodyne detection of light and feedback on atoms, was used for creation of entanglement of two ensembles coupled to a common mode of light, for demonstration of quantum memory and teleportation of states of light to an atomic ensemble [9].

On the other hand, in a system of a nanomechanical oscillator coupled to a coherently driven optical cavity an effective Hamiltonian

$$H_{mc} = \omega_m a_m^\dagger a_m - \delta a_c^\dagger a_c + g(a_m + a_m^\dagger)(a_c + a_c^\dagger), \quad (2)$$

can be realized, where operators $[a_m, a_m^\dagger] = 1$ describe the mechanical oscillator, ω_m is its frequency and δ is the laser detuning from the cavity resonance. For $\delta = -\omega_m$ an interaction of the type H_{BS} can be tuned to resonance and lies at the heart of laser cooling of mechanical oscillators [2, 3], while for $\delta = \omega_m$ a Hamiltonian of the type H_{DC} becomes effective and gives rise to entangling interactions studied in [10]. For resonant drive $\delta = 0$, we arrive at a Hamiltonian, which is formally equivalent to H_{ac} , and was used for feedback cooling of the mechanical resonator [11].

Given the striking parallels between the two systems and the rich toolbox of interactions and measurements, interfacing them seems to be a promising endeavor. We show here that interactions (1) and (2) can indeed be favorably combined and develop a quantum protocol for the implementation of Bell-, and QND measurements, and thus for the generation of entangled states and teleportation in the coupled nano-oscillators - atomic system. We take advantage here of the fact that an inverted harmonic oscillator can be realized in (1) (for $\Omega < 0$), something which is obviously not possible in (2). Remarkably, the present protocol does *not* require ground state cooling of the nanomechanical resonator and avoids holding atoms close to surfaces or inside cavities. This has to be seen in contrast to other theoretical studies on coupling of atomic ensembles to nanoresonators [12, 13]. Treut-

lein et al. [12] suggest to use direct, Zeeman coupling of an ensemble to a nanoresonator carrying a magnetic tip for observing cantilever induced spin flips, and Genes et al. [13] perform steady state entanglement studies of a coupled atomic ensemble - cavity mode - nanoresonator system.

We start with derivation of the opto-mechanical (2) and the light-ensemble interaction (1). Based on this, we show how they can be combined, as shown in Fig.1a, to realize a QND-Bell measurement. As an application we briefly discuss a protocol for teleportation of quantum states, e.g. the ground state, of an atomic ensemble onto the mechanical resonator. We conclude with experimental case studies for two possible setups.

In the opto-mechanical system the fundamental interaction is due to radiation pressure [14] or dispersive forces [5]. In either case the interaction is $V = g_0 a_c^\dagger a_c X_m$, where $g_0 = (x_0/L)\omega_c$ and x_0 is the mechanical oscillator ground state spread, L the cavity length and ω_c its frequency. If the cavity is driven by a resonant pulse of duration $\tau \gg 1/\gamma_c$, where γ_c is the cavity decay rate, and power $P = N_{\text{ph}}\hbar\omega_c/\tau$ containing a total number N_{ph} of photons, a steady state amplitude $\alpha = \sqrt{N_{\text{ph}}/\tau\gamma_c}$ builds up and the dynamics can be linearized giving an effective interaction $V_{\text{eff}} = gx_c X_m$ as in (2), where x_c, p_c describe fluctuations of the cavity field, see [10]. From Hamiltonian (2) the evolution is given by

$$\dot{x}_c = -\gamma_c x_c - \sqrt{2\gamma_c} x_{\text{in}}, \quad \dot{p}_c = -\gamma_c p_c - \sqrt{2\gamma_c} p_{\text{in}} - gX_m,$$

where $[x_{\text{in}}(t), p_{\text{in}}(t')] = i\delta(t-t')$ denotes vacuum noise. Assuming $\gamma_c \gg g, \omega_m$ we adiabatically eliminate the cavity mode and arrive at the cavity input-output relations [15]

$$x_{\text{out}} = -x_{\text{in}}, \quad p_{\text{out}} = -p_{\text{in}} - g\sqrt{2/\gamma_c}X_m. \quad (3)$$

The output field will contain Stokes- and Antistokes-sidebands at $\pm\omega_m$, which are correlated to the mechanical resonator. It is interesting to note that the number of sideband-photons generated in a time τ is roughly on the order of $\sim g^2\tau/\gamma_c = (g_0/\gamma_c)^2 N_{\text{ph}} \simeq 1$ for realistic parameters as given below.

As indicated in Fig.1a the beam leaving the cavity interacts with an ensemble of N_{at} atoms, with a relevant level scheme shown in Fig.1b. This configuration gives rise to Faraday interaction [16], which is described by a Hamiltonian $H \propto J_z S_z$ for atoms in a cavity supporting two polarization modes $[a_i, a_j^\dagger] = \delta_{i,j}$ ($i, j = x, y$). Here $J_z = \sum_{i=1}^{N_{\text{at}}} \sigma_z^i/2$ is the collective ground state spin, with Pauli matrix σ_z^i for atom i , and $S_z = -i(a_x^\dagger a_y - a_y^\dagger a_x)/2$ is a Stokes vector component. Assuming all atoms are initially polarized along x , i.e. $\langle J_x \rangle = N_{\text{at}}/2$, it is possible to introduce scaled variables $X_a = J_y/\sqrt{\langle J_x \rangle}$, $P_a = J_z/\sqrt{\langle J_x \rangle}$ fulfilling approximately canonical commutation relations, $[X_a, P_a] = iJ_x/\langle J_x \rangle \simeq i$. Correspondingly, the free Hamiltonian describing Zeeman splitting

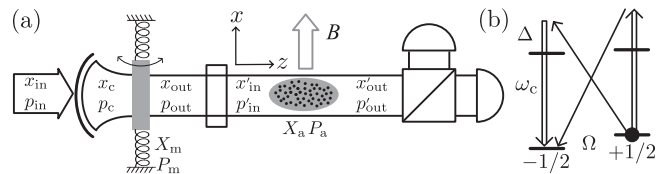


FIG. 1: (a) Schematic of setup: Light interacts first with a mechanical oscillator and then with an atomic ensemble, in a magnetic field B , before it is subject to homodyne detection. A filter (box) phase shifts the coherent carrier and rotates its polarization with respect to the quantum fields in the sidebands, see text. (b) Atomic level scheme: Atoms are pumped into state $+1/2$ of a spin $j = 1/2$ ground state, Zeeman split by Ω . Light of frequency ω_c , propagating along z and linearly polarized along x (double arrows) drives a $j = 1/2 \rightarrow j' = 1/2$ transition, off resonantly with a detuning $\Delta \gg \Omega$. Via virtual transitions y -polarized photons are generated (or absorbed) at sideband frequencies $\omega_c \pm \Omega$ (thin arrows) along with atomic transitions $+1/2 \leftrightarrow -1/2$.

is $H_0 = \Omega J_x = -(\Omega/2)(X_a^2 + P_a^2)$, where the minus sign is due to the fact that atoms are pumped to the energetically higher lying state cf. Fig.1b. For light we assume a large coherent amplitude for x -polarization, $\langle a_x \rangle = \alpha$, such that $S_z \simeq -i\alpha(a_y - a_y^\dagger)/2 \propto p_c$. Overall, we arrive at the Hamiltonian (1). As before, adiabatic elimination of the cavity mode will yield input-output relations

$$x'_{\text{out}} = x'_{\text{in}} + \kappa\sqrt{2/\tau}P_a, \quad p'_{\text{out}} = p'_{\text{in}}.$$

Detailed descriptions [16] show that these equations still hold for atomic ensembles interacting with a propagating pulse in free space. In this case τ is the pulse length and $\kappa^2 = (\sigma\Gamma/A\Delta)^2 N_{\text{at}}N_{\text{ph}}$, where Γ is the spontaneous decay rate, Δ the detuning, σ the scattering- and A the beam-cross section). Also here the number of photons scattered into sideband modes, cf Fig.1b, is approximately $\sim \kappa^2 \simeq 1$ for realistic parameters.

We now assume that – in the sense of cascaded quantum systems [15] – the cavity output provides the input to the light-atoms interaction such that $x'_{\text{in}} = -p_{\text{out}}$ and $p'_{\text{in}} = x_{\text{out}}$. This requires, firstly, that the coherent pulse at frequency ω_c is rotated in polarization by 90° relative to its sideband components at $\omega_c \pm \omega_m$ and, secondly, that the polarization components are phase shifted by $\pi/2$ relative to each other. This can be achieved by separating the optical carrier and the sidebands with an auxiliary optical cavity and then performing the required rotations and shifts. Note that because the sidebands will be measured by homodyning with the carrier, the extinction ratio for the carrier at a percent level is sufficient. Alternatively, birefringency can be introduced in the nanomechanical cavity by, for example, a controlled stress of a mirror. As will soon become clear, an important requirement in the present protocol is to choose parameters such that

$$\kappa\sqrt{2/\tau} = g\sqrt{2/\gamma_c}, \quad (4)$$

which can be fulfilled experimentally as indicated below. Under these conditions the overall input-output relations become

$$x'_{\text{out}} = p_{\text{in}} + \kappa\sqrt{2/\tau}(X_{\text{m}} + P_{\text{a}}), \quad p'_{\text{out}} = -x_{\text{in}}. \quad (5)$$

Thus light reads out the combined so-called EPR (for Einstein-Podolsky-Rosen) variable $X_{\text{m}} + P_{\text{a}}$ [17]. In the next paragraph we will show that this in fact provides a QND measurement [18] of a commuting pair of EPR variables, as in [9] for two atomic ensembles.

To see this, we need to study the evolution of mechanical and collective spin variables. From the discussion above it follows straight forwardly that the mechanical oscillator evolves as,

$$\dot{X}_{\text{m}} = \omega_{\text{m}}P_{\text{m}}, \quad \dot{P}_{\text{m}} = -\omega_{\text{m}}X_{\text{m}} - gx_{\text{c}} = -\omega_{\text{m}}X_{\text{m}} + g\sqrt{\frac{2}{\gamma_{\text{c}}}}x_{\text{in}},$$

where in the last equality the cavity mode was again adiabatically eliminated. In these equations we neglect the decay of the oscillator. We assume the whole protocol happens on a time scale $\sim \tau$ such that $\tau\gamma_{\text{m}}\bar{n}_{\text{th}} \ll 1$, where γ_{m} is the mechanical damping rate and $\bar{n}_{\text{th}} = k_{\text{B}}T/\hbar\omega_{\text{m}}$ is the mean occupation in thermal equilibrium at temperature T . If this condition is met decay can be treated perturbatively, as will be done below. Transverse atomic spin components evolve as

$$\dot{X}_{\text{a}} = -\Omega P_{\text{a}} + \frac{\kappa}{\sqrt{\tau}}p'_{\text{in}} = -\Omega P_{\text{a}} - \kappa\sqrt{\frac{2}{\tau}}x_{\text{in}}, \quad \dot{P}_{\text{a}} = \Omega X_{\text{a}}.$$

Decoherence due to spontaneous emission can be kept small for sufficient optical depth [16] and will be included perturbatively further below. Using again condition (4) and taking in addition $\omega_{\text{m}} = \Omega$, we finally arrive at

$$\frac{d}{dt}(X_{\text{m}} + P_{\text{a}}) = \Omega(P_{\text{m}} + X_{\text{a}}), \quad \frac{d}{dt}(P_{\text{m}} + X_{\text{a}}) = -\Omega(X_{\text{m}} + P_{\text{a}}).$$

In this closed set of equations of motion for commuting EPR observables $P_{\text{m}} + X_{\text{a}}$ and $X_{\text{m}} + P_{\text{a}}$ back action effects of light are canceled out. This establishes the QND character of the present interactions.

The QND signal lies essentially in the Fourier components at frequency Ω of the in-phase component x'_{out} . For the normalized observables $x_{\text{out}}^{\text{cos}} = \sqrt{2/\tau} \int_0^{\tau} dt \cos(\Omega t)x'_{\text{out}}(t)$ and $x_{\text{out}}^{\text{sin}} = \sqrt{2/\tau} \int_0^{\tau} dt \sin(\Omega t)x'_{\text{out}}(t)$ one readily finds the input-output relations,

$$x_{\text{out}}^{\text{cos}} = x_{\text{in}}^{\text{cos}} + \kappa(P_{\text{m}} + X_{\text{a}})_{\text{in}}, \quad x_{\text{out}}^{\text{sin}} = x_{\text{in}}^{\text{sin}} + \kappa(X_{\text{m}} + P_{\text{a}})_{\text{in}}, \quad (6)$$

with appropriate definitions for the in-components $x_{\text{cos}(\text{sin})}^{\text{in}}$. We assume here $\Omega\tau \gg 1$ such that cosine and sine components can be taken as independent variables.

This QND measurement leaves the mechanical resonator and the collective spin in a state, where the EPR operators $P_{\text{m}} + X_{\text{a}}$ and $X_{\text{m}} + P_{\text{a}}$ have a reduced uncertainty as compared to their initial variance, *conditioned*

on the respective measurement results $\xi_{\text{cos}(\text{sin})}$ of $x_{\text{cos}(\text{sin})}^{\text{in}}$. An unconditionally reduced variance can be achieved by a feedback operation on the atomic spin, e.g. via fast rf pulses causing appropriate tilt of the spin, generating a displacement $X_{\text{a}} \rightarrow X_{\text{a}} - g\xi_{\text{cos}}$, $P_{\text{a}} \rightarrow P_{\text{a}} - g\xi_{\text{sin}}$ with a suitable gain g . In the ensemble average this generates a state, whose statistics is described by the input-output relations [19]

$$\begin{aligned} (P_{\text{m}} + X_{\text{a}})_{\text{out}} &= (P_{\text{m}} + X_{\text{a}})_{\text{in}} - gx_{\text{out}}^{\text{cos}} \\ &= (1 - g\kappa)(P_{\text{m}} + X_{\text{a}})_{\text{in}} - gx_{\text{in}}^{\text{cos}}, \\ (X_{\text{m}} + P_{\text{a}})_{\text{out}} &= (1 - g\kappa)(X_{\text{m}} + P_{\text{a}})_{\text{in}} - gx_{\text{in}}^{\text{sin}}. \end{aligned} \quad (7)$$

Our aim is to minimize this variance with respect to the feedback gain g . As initial state of the systems, we assume vacuum for light modes, the ground (fully polarized) state for the collective spin and an initial thermal occupation \bar{n}_i for the mechanical resonator. In thermal equilibrium $\bar{n}_i = \bar{n}_{\text{th}}$, but in principle \bar{n}_i can be reduced by initial laser cooling [2, 3]. The minimal variance

$$\Delta(P_{\text{m}} + X_{\text{a}})_{\text{out}}^2 + \Delta(X_{\text{m}} + P_{\text{a}})_{\text{out}}^2 = \frac{2}{\frac{1}{(1+\bar{n}_i)} + 2\kappa^2} \quad (8)$$

is the main result of this paper.

Its importance is due to the fact that the inequality $\Delta(P_{\text{m}} + X_{\text{a}})_{\text{out}}^2 + \Delta(X_{\text{m}} + P_{\text{a}})_{\text{out}}^2 < 2$ constitutes a sufficient – and for the present case of Gaussian states also necessary – condition for entanglement between the mechanical resonator and the atomic ensemble [20]. Remarkably, *there is no fundamental limit on observable entanglement due to initial thermal occupation \bar{n}_i* , as $2[(1 + \bar{n}_i)^{-1} + 2\kappa^2]^{-1} \leq \kappa^{-2}$. Thus, even for moderate values of $\kappa \gtrsim 0.5$, achievable as outlined below, the present protocol provides an entangled state independent of the initial thermal occupation of the nanomechanical resonator. Some intuition for this effect can be gained from the observation, that a strong QND measurement $\kappa \rightarrow \infty$ asymptotically realizes a projective measurement on the system, collapsing it to a pure, maximally entangled EPR state. There are of course other limiting factors, which we will discuss below. For now, we just want to point out that thermal excitations merely restrict the tolerable duration of the protocol and the life time of entanglement.

The entanglement created here can be used in particular for a teleportation protocol for quantum states carried by an atomic ensemble onto the mechanical resonator. Assume we first prepare an entangled state characterized by Eq.(8) and a second, additional atomic ensemble in a coherent state with amplitudes $\langle X_{\text{a}2} \rangle$, $\langle P_{\text{a}2} \rangle$. A QND Bell measurement on the two atomic ensembles is performed, as demonstrated in [9]. The result of such a measurement is essentially given by Eqs.(6) with $(P_{\text{m}} + X_{\text{a}})$ and $(X_{\text{m}} + P_{\text{a}})$ replaced by $(P_{\text{a}} + X_{\text{a}2})$ and $(X_{\text{a}} + P_{\text{a}2})$ respectively. An appropriate feedback, via e.g. piezoelectric or radiation pressure displacement of the mirror,

will complete the teleportation and generate a state,

$$X_m^{\text{telep}} = X_m + g[x_{\text{in}}^{\text{cos}} + \kappa(P_a + X_{a2})] = X_m + P_a + X_{a2},$$

$$P_m^{\text{telep}} = P_m + g[x_{\text{in}}^{\text{sin}} + \kappa(X_a + P_{a2})] = P_m + X_a + P_{a2}.$$

Here κ and g denote strength of QND interaction and feedback gain in the Bell measurement on the two atomic ensembles. The second equalities of both lines are valid in the asymptotic limit $\kappa \rightarrow \infty$, $g \rightarrow 0$ while $\kappa g = 1$, which essentially requires a large optical depth[16]. Amplitudes are thus transmitted correctly, $\langle X_m \rangle = \langle X_{a2} \rangle$ etc., and the amount of added noise, $\Delta(X_m + P_a)^2$ etc., follows from Eq.(8). For $\kappa \simeq 1$ this is approximately one unit of vacuum noise in each variable, corresponding to a fidelity of 2/3. As a side mark, we note that this implies the intriguing possibility to cool a mechanical resonator by teleporting the ground state onto it. A similar observation was already made by Mancini et al. [10], albeit in a different setting.

We turn to the discussion of losses and decoherence. The dominant impairing effects, are (i) number mismatch in Eq.(4), (ii) loss of light and detection inefficiency, spontaneous emission in light-atom interactions and (iii) thermalization of the mechanical oscillator. Ad (i), it is straight forward to derive, that a nonzero mismatch $\epsilon = (\kappa/\sqrt{\tau} - g/\sqrt{\gamma_c})/(\kappa/\sqrt{\tau} + g/\sqrt{\gamma_c})$ will give rise to additional terms in the variance of EPR variables (8) scaling in leading order as $[\epsilon\kappa(\bar{n}_i + 2)]^2$ i. For $\kappa \gtrsim 1$ a mismatch of $\epsilon \simeq 1/10\bar{n}_i$ is tolerable. Effects due to processes (ii) and (iii) can be treated perturbatively as linear losses, as we exemplify for damping of the resonator: In the course of the interaction the state of the resonator will undergo damping at a rate γ_m and provided $\gamma_m\tau \ll 1$ e.g. Eq.(7) will generalize to

$$(P_m + X_a)_{\text{out}} = (\sqrt{1 - \gamma_m\tau}P_m + X_a)_{\text{in}} + \sqrt{\gamma_m\tau}f_{X_m} - g x_{\text{out}}^{\text{cos}},$$

where f_{X_m} is a Langevin operator for thermal noise, $\langle f_{X_m}^2 \rangle = (\bar{n}_{\text{th}} + 1)$. The variance will thus receive an additional term $\gamma_m\tau(\bar{n}_{\text{th}} + 1)$, such that we have to require $\tau \ll 1/\gamma_m\bar{n}_{\text{th}} = Q_m\hbar/k_B T$ for a quality factor $Q_m = \omega_m/\gamma_m$. In a similar vein one can treat losses (ii), which have the advantageous property that the corresponding noise is, to a good approximation, vacuum noise. This will reduce, but not remove the entanglement created in this protocol.

The suggested protocol can be realized with current technology. We consider two possible setups, in which the nanomechanical resonator is used either as one of the mirrors of a Fabry-Perot cavity [2] or as a dispersive element in a rigid cavity [5]. Assuming that $\kappa \simeq 1$ and that condition (4) can be matched within an error of $\epsilon = 0.01$ we need $\bar{n}_i \lesssim 30$. This can be achieved for high ω_m at dilution refrigerator temperatures, cf. Naik et al. [2] or, in case of lower ω_m or higher bath temperatures, by applying additional modest laser cooling. As an example for the two cases, we assume a moving micromirror with

$\omega_m/2\pi = 5$ MHz, mass $m = 10^{-12}$ kg and quality factor $Q_m \geq 3 \times 10^6$ operated at $T = 1$ K (resulting in $\bar{n}_i \approx 4000$, which requires laser cooling by a factor of 150), and a small (dispersively coupled) nanomembrane with $\omega_m/2\pi = 30$ MHz, $m = 10^{-14}$ kg and $Q_m \geq 10^5$ operated at $T = 0.04$ K ($\bar{n}_i \approx 30$). Mechanical quality and temperature limit the interaction time to $\tau \ll 20 \mu\text{s}$, which is in principle sufficient for entanglement of room temperature atoms and certainly enough in the case of cold atoms. For the laser-cooled micromirror (4) can be achieved with a finesse $F = 4500$ and power $P = 100 \mu\text{W}$. Adiabatic elimination of the cavity mode finally poses an upper bound $L \leq 300 \mu\text{m}$ on the cavity length. For the nanomembrane a modest finesse $F = 1100$ is already sufficient at a pump power $P = 100 \mu\text{W}$ and cavity length $L \leq 250 \mu\text{m}$.

In conclusion, given the rich toolbox of well established interactions and measurement techniques, we expect the proposed interface to open a new avenue in atomic ensemble physics and nanomechanics.

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